

THE
LONDON, EDINBURGH, AND DUBLIN
PHILOSOPHICAL MAGAZINE
AND
JOURNAL OF SCIENCE.

[SEVENTH SERIES.]

JUNE 1931.

XCVII. *Measurement of the Accession to Inertia of a Vibrating Diaphragm.* By N. W. McLACHLAN, D.Sc., M.I.E.E., F.Inst.P.*

SYNOPSIS.

1. Five Methods of Measuring the Accession to Inertia (M_1).
2. Experimental Arrangements and Precautions necessary to attain Accuracy.
3. Bridge Method of Measuring Inductance.
4. Experimental Data for Conical Diaphragm, illustrating Method 3. Comparison with Rigid Disk Theory.
5. Experiments showing the Influence of a Rubber Surround on the Measurements of M_1 .
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Appendix.

Measurement of M_1 in System with Elastic Constraint.

ABSTRACT.

THE paper is concerned with methods of measuring the accession to inertia of vibrating diaphragms.

Five methods are described, each necessitating the measurement of the inductance of a coil situated in a magnetic field. The diaphragm is driven by virtue of the reaction between this field and that of an alternating current in the coil. The inductance is measured with the coil fixed

* Communicated by the Author.

(L_0) and free to move in air (L_1) and *in vacuo* (L_v). In the free condition the value of inductance depends upon the effective mass of the moving system. Knowing L_0 , L_1 , L_v , and m , the natural mass of the system, the accession to inertia M_1 can be calculated. The results obtained for a conical diaphragm are in good agreement with those computed from Rayleigh's formula for a rigid disk. It is shown that with a limited size of baffle the accession to inertia decreases with the frequency. Numerous precautions to attain experimental accuracy are given, and experiments showing the necessity for the diaphragm to move as a whole are quoted.

1. Methods of Measurement.

IT has been shown in a previous paper*—confirming a prediction of the late Lord Rayleigh—that the accession to inertia (M_1) of a diaphragm vibrating in air is a reality. The object of the present contribution is to describe methods of measuring this quantity which are superior to that cited in the previous paper, and to give results obtained using conical paper diaphragms driven by circular coils† immersed in a radial magnetic field.

Method 1.

From Phil. Mag. pp. 1017 & 1033, June 1929, and p. 36, Jan. 1931, the reactance of the coil in motion driving a rigid disk is given by

$$X = \omega L_0 - 1/\omega C_m = \omega L_1, \quad \dots \quad (1)$$

where $C_m = m/C^2 \cos^2 \phi (1 - \cos^2 \theta) = m/C_1^2 \ddagger$.

$m = M + M_1$ = natural mass of vibrating system
+ accession to inertia.

$C = 2\pi hrH$ = length of wire on coil \times mean flux density in air gap.

ϕ = phase angle between magnetizing and total current in coil, due to iron loss.

$\cos \theta \begin{cases} = \text{acoustic power factor of diaphragm.} \\ = (\text{acoustic pressure} + \text{losses})/\text{driving force.} \end{cases}$

* Phil. Mag. xi. p. 35 (Jan. 1931).

† Obviously the coil can have any convenient shape.

‡ This expression only holds when there is no mechanical constraint. See Appendix for the latter condition. For convenience throughout this paper C has been written for C_1 , since they are nearly equal.

L_0 = inductance of coil when fixed in magnet.

L_1 = inductance of coil in motion in magnetic field. Below the electromechanical resonance frequency it is negative.

From the preceding by substitution and transposition we obtain the accession to inertia,

$$M_1 = C^2/\omega^2(L_0 - L_1) - M^*, \quad . \quad . \quad . \quad (2)$$

a relationship which holds for vibration in any fluid or gas. In *vacuo* $M_1 = 0$ and $\cos \theta$ alters because the acoustic pressure on the diaphragm vanishes. Since $\cos^2 \theta$ is usually small compared with unity during vibration in *air* it will be neglected. Moreover, C^2 is substantially the same in air and in *vacuo*. Moreover, in the latter case

$$\frac{C^2}{\omega^2(L_0 - L_v)} \doteq M, \quad . \quad . \quad . \quad (3)$$

by aid of which C^2 can be obtained without knowing H , $\cos \phi$, n , r , or the axial field distribution in the gap.

$$\text{From (3)} \quad C^2 = M\omega^2(L_0 - L_v),$$

where v is a subscript denoting measurement of L_1 in *vacuo*.

Substituting this value of C^2 in expression (2), we find that

$$M_1 = \frac{M\omega^2(L_0 - L_v)}{\omega^2(L_0 - L_1)} - M,$$

i. e., the accession to inertia equals

$$\frac{M(L_1 - L_v)}{(L_0 - L_1)}. \quad . \quad . \quad . \quad (4)$$

Obviously there is no necessity to measure C^2 , and the accession to inertia, in cases where $\cos \theta$ is negligible, is found by weighing the diaphragm to get M , ascertaining the inductance of the coil when fixed (L_0) †, whilst in motion in air (L_1), and in *vacuo* (L_v).

Method 2.

This is a variation of method 1, using two identical reproducers—if such a thing is feasible,—one in air, the

* Alternatively $m = M + M_1 = C/\omega^2(L_0 - L_1)$.

† This can conveniently be found by aid of an identical auxiliary reproducer in which the coil is permanently glued or fixed to the magnet.

other *in vacuo*. With the magnetic fields on and both diaphragms free to move ($L_1 - L_v$) can be measured by connecting the two coils in opposition. With one diaphragm in air free to move as above, and the other fixed *in vacuo* * or the magnetic field off, ($L_1 - L_0$) can be measured, from which we obtain M_1 as in expression (4). *In vacuo* there is no acoustic pressure and the necessary condition is satisfied if the two coils are electrically identical and the natural masses of the moving systems are equal. Thus any suitable structure, such as a small box capable of holding weights, can be fixed to the coil *in vacuo*, and the mass adjusted until it equals that of the system in air. It is assumed, of course, that the diaphragm under test moves as a whole.

Method 3.

When vacuum equipment is not available the necessary conditions can be simulated by using the moving coil without the diaphragm, this being suspended by elastic threads to have a very low natural frequency (field off). The absence of the diaphragm means that the accession to inertia M_1 is substantially zero. Thus expression (3) is applicable and can be written

$$C^2 = M_c \omega^2 (L_0 - L_c), \quad . \quad . \quad . \quad . \quad (5)$$

where the subscript c denotes use of the coil alone. Substituting for C^2 in (2) we obtain

$$M_1 = \frac{M_c (L_0 - L_c)}{(L_0 - L_1)} - M. \quad . \quad . \quad . \quad . \quad (6)$$

Method 4.

This is a variation of method 3 and is also useful where vacuum equipment is not available. Two identical reproducers are necessary, but the diaphragm of one is replaced by a small mass whose value is equal to the natural mass of the diaphragm under test. If the vibrating area is very small the accession to inertia is negligible, so that expression (4) is applicable.

Where the natural mass of the reproducer is indefinite, due to the presence of a surround whose amplitude varies from the diaphragm to the outer clamping ring, two identical models must be used, one in air, the other *in vacuo*. Care

* This condition can also be secured by the aid of a third coil permanently glued to a magnet in air.

must be exercised, however, that resonances do not influence the result (see Section 5).

Method 5.

This is the method described in a previous paper*. Measurements are made of L_0 , L_1 , M , and H , the air gap flux density. The latter is found, as shown elsewhere†, by the aid of a differential search coil. $\cos \phi$ can be ascertained from measurements of L_0 and R_0 . Its value lies between 0.95 and 1 for the apparatus used in these tests. However, as a first approximation both $\cos \phi$ and $\cos \theta$ can be neglected. Strictly H is the root mean square value, and this involves a knowledge of the axial distribution of flux in the gap. Again, we can neglect this to obtain an approximation, which is computed using expression (2) above.

2. Experimental Arrangements and Precautions necessary to attain Accuracy.

The general arrangement of the diaphragm and magnet used for the experiments is illustrated diagrammatically in fig. 1. The edge of the diaphragm is reinforced to eliminate radial modes at the lower frequencies, whilst it is supported by four or six elastic threads, so that the natural frequency axially, in the absence of the magnetic field, does not exceed 3 cycles per second. In some cases the moving coils were wound with 40 turns of 28 d.w.s. wire and bakelized. Other coils had 1000 turns of 47 enamelled wire, but considerable care is required in winding such coils to avoid short-circuited turns‡. Also wire of this gauge with good enamel is difficult to obtain. Any number of turns will serve the purpose, and it is usual to adopt an inductance which can be measured by aid of a Campbell Mutual Standard. Where the value of $\cos^2 \theta$ is small enough to warrant high accuracy the following precautions should be taken:—

1. The temperature, pressure, and humidity of the air during the measurements must be known, so that the density ρ_a can be calculated accurately.

2. The position of the moving coil in the magnet, the temperature of the coil, bridge and leads, and the current

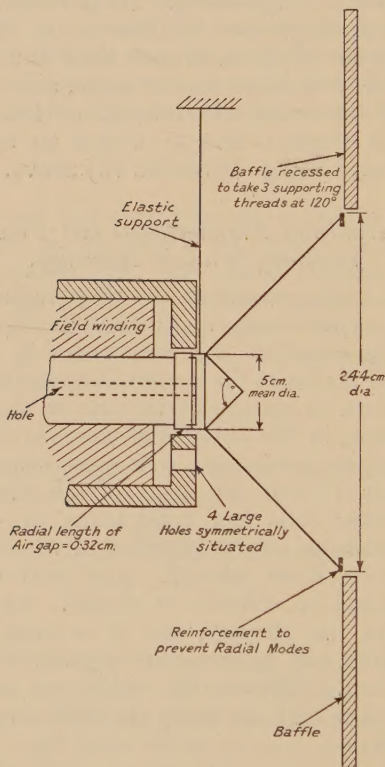
* Phil. Mag. xi. p. 36 (Jan. 1931).

† 'Wireless World and Radio Review,' p. 600 (Nov. 26th, 1930).

‡ These can be detected by inductance and resistance measurements at, say, 1000 cycles.

in the magnetizing coil should be identical for tests in air and *in vacuo*. Owing to the large loss in the magnetizing coil the temperature rise is appreciable and affects the moving coil. The condition can be checked by taking resistance measurements by a d.c. bridge. A relay or a switch should be arranged to open the coil just before the

Fig. 1.



Diagrammatic sketch illustrating arrangement of diaphragms used for the determination of M_1 , the accession to inertia. The presspahn reinforcement prevents radial modes. Angle of cone 90° .

magnetic field is switched off or on. This prevents the coil being disturbed due to impulsing.

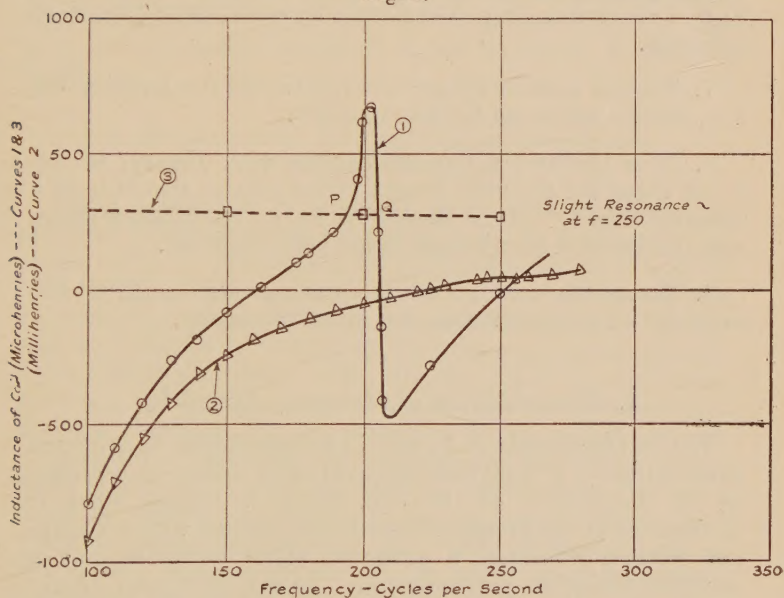
3. The leads from the moving coil to the terminal block and the suspension of the diaphragm should be arranged either to have a calculable or a negligible mass. The

natural frequency in the absence of the magnetic field should not exceed 3~ per sec.

4. The mass of the diaphragm should be found immediately after the measurement of L_1 in air to avoid variations due to absorbed air and moisture. Due to expulsion of the latter the mass *in vacuo* may be slightly less than in air.

5. The diaphragm must move substantially as a whole at the frequencies used, *i. e.*, the tests must be conducted below the "break-up" frequency. This should be ensured by

Fig. 2.



Curve 1 = inductance (L_1) of 40-turn coil in motion, driving diaphragm as shown in fig. 1, but with presspahn ring 2.5 cm. wide glued to periphery. There are three frequencies for which $L_1=0$, this being due to the elasticity of the presspahn ring.

Curve 2 = inductance (L_1) of 1000-turn coil in motion, driving diaphragm as shown in fig. 1. There is only one electromechanical resonance, *i. e.*, $L_1=0$ only once. A slight resonance occurs at 250 cycles per second.

Curve 3 = inductance (L_0) of 40-turn coil fixed. It cuts the motional curve at two points, P Q, giving $L_1-L_0=0$ twice. Consequently M_1 ascalculated = ∞ twice.

stroboscopic observations and by a smooth frequency L_1 curve (see fig. 2, curve 2). A flat disk of balsa wood with

stiffening ribs and a coil with a long stiff neck to keep the disk well away from the magnet might be useful for comparison with theory. $\cos^2 \theta$ could be introduced here.

6. If a reproducer is designed to work with a definite size of baffle the latter should be used for tests in air. For comparison with theory, where an infinite plane wall is contemplated, the baffle should be as large as possible, to prevent interference between the two sides of the diaphragm. A non-resonant non-absorbent baffle 15 feet square is suggested for frequencies down to 100 cycles, and the side of a large wall for lower values. The tests should be made in free air or in a draped enclosure to avoid the influence of reflexion.

7. The coil must be adequately rigid at the test frequencies, *i. e.*, there must be no flexural modes*.

8. It is preferable, for comparison with theory, to use a coil about 0.5 the axial length of the gap, so that it is in a uniform field†. There should be no short-circuited turns, and the use of a transformer is not recommended.

9. The testing current in the moving coil should be the same for all measurements at a given frequency.

3. *Bridge Method of Measuring L_0 and L_1 .*

The measurements of L_0 and L_1 are conducted by an equal ratio bridge or a Heaviside unequal ratio bridge—according to the inductance of the coil, which is largest when it is negative at low frequencies—in conjunction with a Campbell Mutual Standard, a powerful valve oscillator, and a sensitive vibration galvanometer. In the particular reproducing instrument used for these experiments the coil “fixed” condition was adequately simulated by short-circuiting the magnet winding on itself‡. The remanent field was very small and the mutual inductance between the magnetizing (fixed) and moving coils negligible. This saves endless trouble fixing the coil and the diaphragm, and avoids the possibility of shifting the position of the coil and of temperature variations for the free and fixed conditions at a given frequency. Moreover, in what follows L_0 is the

* See Phil. Mag. xi. p. 30 (Jan. 1931).

† Phil. Mag. *loc. cit.* p. 39.

‡ The difference was about 1 per cent.

inductance with the field off*, the reading being taken immediately before and after that of L_1 with the field on.

The value of L_1 is zero at the electromechanical resonance frequency †, and below this it is negative owing to the capacity action concomitant with the motional back e.m.f. Care must be exercised to ensure that no resonances of any description occur at frequencies where measurements of L_1 are made, or serious errors will arise. In certain cases $L_1=0$ at several frequencies, as shown in curve 1, fig. 2.

This particular result is due to the elastic properties of a piece of presspahn 2.5 cm. wide glued to the edge of the diaphragm to prevent radial modes. An adequate stiffness is given by thicker presspahn 0.7 cm. wide. There is then only one frequency, for which $L_1=0$, as shown in curve 2, fig. 2.

4. *Experimental Data for Conical Diaphragm illustrating Method 3.*

Using method 3 a series of tests were conducted at different frequencies on a diaphragm whose dimensions are shown in fig. 1. The results are set forth in Tables I. and II.

There is a fair agreement between the calculated and experimental values of M_1 at 150 and 200 cycles. In this respect it should be noted that the calculated results refer to a *flat* disk in an *infinite* baffle, whereas those by experiment refer to a *conical* diaphragm in a baffle of limited dimensions. Strictly, therefore, the comparison is not between theory and practice for a rigid disk, but between two dissimilar shapes (cone and disk), to show the existing resemblance. So far as could be seen stroboscopically, the diaphragm moved as a whole at these frequencies. The value of M_1 is down at 100 cycles, owing to the size of the baffle being inadequate to prevent appreciable interference between the two sides of the diaphragm. As the frequency is reduced below 100 cycles the accession to inertia slowly decreases with a baffle 6 feet square.

Readings were also obtained in the absence of the baffle. In the neighbourhood of 200 cycles the effect was marked, *e.g.*, for a certain diaphragm at 185 cycles $M_1=12$ gm. with

* For any particular magnet and air gap tests should be made to ascertain whether L_0 is the same when the coil is fixed and the field on, and when free with the field short-circuited. The necessary condition would not hold where the magnetic material had a high remanence.

† See Phil. Mag. p. 1033 (June 1929).

and 8.8 gm. without the baffle*. In the 50 cycle region the effect of the baffle is appreciably less than at 200 cycles.

TABLE I.

Illustrating the method of finding $M_c(L_0 - L_c)$ of expression (6). The effect of the bridge leads cancels out, since L_0 is positive whilst L_c is negative. Turns on coil = 40 of 28 S.W.G. about 2.5 cm. mean radius. Effective mass of coil, leads, and suspension, $M_c = 7.5$ gm.

f . Frequency (cycles per second).	L_0 . Inductance of coil at rest, including bridge leads (microhenries).	L_c . Inductance of coil in motion without diaphragm, including bridge leads (microhenries).	$M_c(L_0 - L_c)$ (gram- microhenries).
100.....	+324	-6.624×10^3	5.21×10^4
150.....	+310	-2.776×10^3	2.31×10^4
200.....	+302	-1.398×10^3	1.275×10^4

TABLE II.

Data illustrating computation of M_1 . Effective mass of coil, diaphragm, leads, etc. = 25 gm. Baffle 6 feet square.

f . Frequency (cycles per second).	L_0 . Inductance of coil at rest, including bridge leads (microhenries).	L_1 . Inductance of coil in motion with diaphragm, including bridge leads (microhenries).	M_1 . Accession to inertia (gm.).	
			Experiment using baffle 6 ft. square.	Calculation assuming infinite baffle.
100.....	+324	-1208	9	12
150.....	+310	-315	12	12
200.....	+302	-34	13	12

since the half wave-length of sound is 12 feet, or about twice the shortest air-path from the centre of one side of the diaphragm to that of the other. I have shown elsewhere †

* The front of the support for the magnet frame acted as a partial baffle, so that removal of the main baffle would not eliminate the baffling entirely.

† In a contribution to the discussion on a paper to be published in the Journal I. E. E. about May or June 1931.

that M_1 for a rigid circular disk in an infinite baffle at low frequencies is *twice* its value when there is no baffle. Under the latter condition M_1 for a sphere vibrating along a diameter is $4/\pi = 1.28$ times that of a flat circular disk, indicating that the shape of the diaphragm is of importance, since it modifies the motion of the fluid. In the present experiments with a cone there is a tendency for M_1 to be in excess of the value for a rigid disk in an infinite baffle in free air. No attempt was made, however, to simulate the latter condition, so that the results must be interpreted as values obtained under certain experimental conditions likely to arise in practice. It must be remembered that a sphere encloses a volume whereas a cone does not. Doubtless a double cone would give a value of M_1 approaching that for a sphere.

The effect of adding the baffle can be seen immediately by increase in the value of L_1 . In a resonating and in a non-resonating combination variation of L_1 is a criterion of changes of mass of the moving system, whether due to elasticity or to accession to inertia. This is illustrated clearly by the abrupt alteration in curve 1 at each side of the resonant frequency where the effective mass is zero. From (10) C_m is infinite at resonance, and from (1) $L_1 = L_0$. Curve 2, fig. 2, gives the values of L_1 for a coil of 1000 turns at different frequencies. The curve is quite smooth up to the electro-mechanical resonance frequency at 225 cycles per second, but a slight resonance occurs at 250 cycles. This shows that—unless peculiarities are revealed stroboscopically—measurements of M_1 below 200 cycles should be reliable.

5. Experiments showing the Effect of a Rubber Annulus supporting the Edge of a Conical Diaphragm.

When a conical diaphragm is mounted on an annular rubber surround, so that the natural axial frequency of the diaphragm moving as a whole thereon is below 20 cycles per second, it might be thought that the accession to inertia could be found by the methods described herein. This, however, is not the case, since the surround itself acts as an auxiliary resonant diaphragm having a frequency well within the audible lower register*.

To illustrate this point concretely some approximate data are given in Table III. In this case method 5 was used, and the accession to inertia calculated from expression 2.

* See Phil. Mag. p. 28 (Jan. 1931).

Starting at 75 cycles, the apparent value of M_1 exceeds the theoretical value for a rigid disk, and increases until it is infinite twice, then falls and rises again. These variations are due to resonance of the surround *per se*, and are of a similar nature to the effect of the presspahn edge shown in fig. 2, curve 1. The L_1 curve (coil in motion) cuts the L_0 curve (coil at rest) in two points, thereby giving $L_1 - L_0 = \text{zero}$ twice. Consequently M_1 , calculated from expression (2), is infinite twice. Moreover, with resonances of this magnitude the procedure for obtaining M_1 is quite invalid.

TABLE III.

Data illustrating the influence of a Resonant Surround on M_1 .

f . Frequency (cycles per second).	M_1 (gm.).	
	Apparent accession to inertia from expression 2.	Rigid disk theory, with infinite baffle.
75.....	22.5	15
100.....	48	15
125.....	infinite	15
127.....	infinite	15
140.....	8	15
200.....	19.5	15

6. Experiments to determine the effect of adding a Mass to a Diaphragm.

An interesting series of experiments was performed to ascertain the mass of a 10 gm. weight added to a diaphragm, as computed from measurements of L_0 and L_1 , using expression (2). When attached (with glue) to the top of the cone halfway between the outer stiffened edge and the coil, the added mass computed from electrical measurement was substantially zero, *i. e.*, the value of L_1 at 80 cycles per second was practically the same with and without the mass. Stroboscopic examination showed that the mass pitched, tossed, and rolled like a ship on a rough sea. The diaphragm simulated the sea, and its dynamical state was so perturbed that the portion in the immediate vicinity of the mass had "broken up." Moreover, by virtue of this phenomenon, the mass had no effect on the electrical coefficients of

the coil. A similar result was obtained when the mass was attached to the centre of a flat paper disk across the mouth of the cone. When the mass was placed on top of the cone near the stiff outer edge, its value as found from electrical measurements agreed fairly well with its actual value. These experiments show how easy it is to make a diaphragm break up locally below its main break-up frequency*, and that under this condition no reliance can be placed on measurements pertaining to the accession to inertia.

When these experiments are performed in air we have from (2)

$$m = \frac{C^2}{\omega^2(L_0 - L_1)},$$

also
$$m + m_1 = \frac{C^2}{\omega^2(L_0 - L_m)}.$$

By eliminating m the added mass is given by

$$m_1 = \frac{m(L_m - L_1)}{(L_0 - L_m)}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (7)$$

where m = mass of diaphragm + accession to inertia as measured previously,

and L_m = inductance with added mass m_1 .

When both experiments (with and without m_1) are conducted *in vacuo* the expression is

$$m_1 = \frac{M(L_{mv} - L_n)}{(L_0 - L_{mv})}, \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (8)$$

M being the mass of the diaphragm and v denoting measurement *in vacuo*.

From (7), substituting for m from (2), we obtain

$$m_1 = \frac{C^2}{\omega^2(L_0 - L_1)} \left[\frac{(L_m - L_1)}{(L_0 - L_m)} \right], \quad \cdot \quad \cdot \quad \cdot \quad (9)$$

but this necessitates measurement of the flux density H to secure C^2 , although neither the mass of the diaphragm nor the accession to inertia need be known.

It will be seen that no stipulation has been made throughout concerning the shape of the diaphragm, so that the methods of measuring the accession to inertia, or, in fact,

* The extra mass of the seam causes an effect of this nature, but it occurs at about 200 to 250 cycles.

any added mass, are applicable universally, with the proviso that the diaphragm moves as a whole. By using smaller diaphragms and lighter coils the method can be extended to frequencies higher than those quoted herein.

APPENDIX.

Accession to Inertia of Rigid Diaphragm with Elastic Constraint.

If the diaphragm moves as a whole, and the system can be treated as a loaded coil spring, we have from Phil. Mag. p. 4, January 1931,

$$C_m = - \frac{[\omega^2 B^2 + (k_1 - \omega^2 m)^2]}{\omega^2 C^2 (k_1 - \omega^2 m)} \dots \dots (10)$$

Assuming the effect of sound radiation, diaphragm, and other losses to be negligible, *i. e.*, that

$$k_1 - \omega^2 m \geq B,$$

which means that ω is well away from the resonance value, we can write

$$\begin{aligned} C_m &= \frac{(\omega^2 m - k_1)}{\omega^2 C^2} \\ &= \frac{1}{C^2} \left(m - \frac{k_1}{\omega^2} \right) \dots \dots \dots (11) \end{aligned}$$

Inserting this value of C_m in (1) we get

$$\begin{aligned} m - \frac{k_1}{\omega^2} &= \frac{C^2}{\omega^2 (L_0 - L_1)}, \\ \text{or} \quad C^2 &= \omega^2 \left(m - \frac{k_1}{\omega^2} \right) (L_0 - L_1) \dots \dots (12) \end{aligned}$$

In vacuo, since $M_1 = 0$, we get

$$C^2 = \omega^2 \left(M - \frac{k_1}{\omega^2} \right) (L_0 - L_v) \dots \dots (13)$$

Equating (12) and (13), we have

$$M_1 = \left(M - \frac{k_1}{\omega^2} \right) \frac{(L_1 - L_v)}{(L_0 - L_1)}, \dots \dots (14)$$

as might have been anticipated from (4), since $M - \frac{k_1}{\omega^2}$ is the apparent mass of the diaphragm. The value of k_1

can conveniently be found by ascertaining the resonance frequency from bridge measurements *in vacuo*. Then $k_1 = \omega^2 M$. In fact if ω_0 is the resonance frequency in air, we have $k_1 = \omega_0^2 (M + M_1)$, giving $M_1 = M \left(\frac{\omega^2}{\omega_0^2} - 1 \right)$ at a frequency $\frac{\omega_0}{2\pi}$. By increasing or decreasing k_1 the value of M_1 could be found at different frequencies.

When a vacuum is not available and C^2 is found by method 3 we have from (12)

$$C^2 = \omega^2 (M_c - k_1/\omega^2) (L_0 - L_c) (15)$$

Equating (12) and (15) we obtain

$$(m - k_1/\omega^2) (L_0 - L_1) = (M_c - k_1/\omega^2) (L_0 - L_c),$$

and since $M + M_1 = m$,

$$M_1 = [(M_c - k_1/\omega^2) (L_0 - L) / (L_0 - L_1)] - (M - k_1/\omega^2). (16)$$

In method (4) $M_c = M$, and expression (16) then reduces to

$$M_1 = (M - k_1/\omega^2) (L_1 - L_c) / (L_0 - L_1). . . (17)$$

ERRATA IN PREVIOUS PAPERS.

Phil. Mag. viii. Supplement, June 1929.

P. 1019, expression (17). For $[\omega^2(k_1 L + BR + C^2) \text{ etc.}]^2$

read $[\omega(k_1 L + BR + C^2) - \omega^3 m L]^2$.

P. 1021, expression (30). For

Dxv etc. read $Dx = v$ etc.

P. 1026, line 2. For

$$\frac{16}{3} a^2 \rho_d \quad \text{read} \quad \frac{16}{3} a^3 \rho_d.$$

P. 1026, expression (41). For

$$\frac{F}{m} \quad \text{read} \quad \left(\frac{F}{m} \right)^2.$$

Phil. Mag. xi. Jan. 1931.

P. 3, expression (1). For

$$\omega^2 B + (k_1 - \omega^2 m)^2 \quad \text{read} \quad \omega^2 B^2 + (k_1 - \omega^2 m)^2.$$

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P. 7, column 2 of Table II. *For*

$$R_m = \frac{C \cos^2 \theta}{B} \quad \text{read} \quad R_m = \frac{C^2 \cos^2 \theta}{B}.$$

P. 13, line 1. *For*

$$\sigma^2 - 1/\sigma \quad \text{read} \quad \sigma^2 - 1/\sigma^2.$$

P. 28, expression (7). *For*

$$f_1 = \left[\frac{1}{2} \frac{T_1}{m_1 l} \right]^{\frac{1}{2}} \quad \text{read} \quad f_1 = \frac{1}{2} \left[\frac{T_1}{m_1 l} \right]^{\frac{1}{2}}.$$

P. 29, line 8. *For*

$$T_1 = \frac{k_0 x}{\theta} \text{ etc.} \quad \text{read} \quad T_1 = \frac{k_0 x}{\theta} = k_0 l, \text{ since } x = l\theta.$$

P. 50, line 11. *Delete* "second main."

P. 53, Table XIII. *For*

$$K_1(z) = 3.20 \text{ when } ka = 4.0 \quad \text{read} \quad K_1(z) = 4.20.$$

January 1931.

XCVIII. *On the Technique of Spectral Intensity Measurement, with Special Reference to Choice of Photographic Plate.* By C. W. W. READ, B.Sc., *Layton Research Student, King's College, London,* and L. W. JOHNSON, M.A., *King's College, London* *.

[Plate XIV.]

Introduction.

AS a preliminary to the development of experimental work on intensities in band spectra we have investigated the relative merits of some different methods of procedure and conditions governing the choice of suitable photographic plates. A number of difficulties were encountered in the course of acquiring the technique—difficulties which must be commonly experienced in this class of work—and the purpose of the present communication is to present our own experiences in the hope that they may be of value to others contemplating this type of work.

* Communicated by the Authors.

Spectroscopic intensity measurements obtained by the use of the photographic plate depend essentially on a principle enunciated in 1899 by Hartmann ⁽¹⁾ to the effect that "two intensities may be considered equal if they produce equal blackening on the same plate for equal exposures of time." This simple fundamental principle cannot be applied directly in practice, and other variable factors, such as exposure time, plate type, intensity, wave-length, etc., make the problem of comparing and contrasting intensities from the blackening produced on a photographic plate a very intricate one.

To define the term blackening as applied to a photographic plate, consider a spectrographic plate which has been used to record monochromatic light. Suppose a beam of light to pass through the developed plate, the intensity of the light transmitted through the line image will be less than that transmitted through the background. If i_0 is the intensity which passes through the background, and i the intensity through the blackened plate, then S , the blackening is defined by

$$S = \log_{10} \frac{i_0}{i}.$$

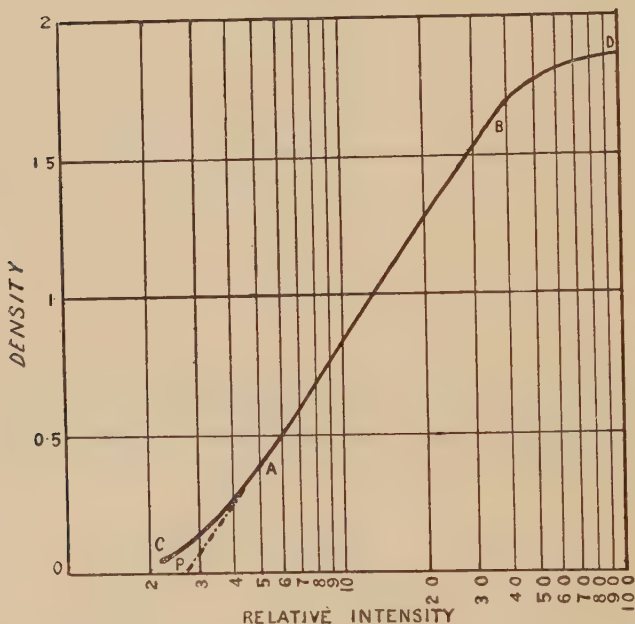
This density of image, S , is a complicated function involving, among other factors, J , the intensity of the light producing the blackening; t , the time of exposure; and λ , the wave-length of the light photographed. To indicate the factors involved we may write

$$S = f(J, t, \lambda, \text{kind and age of plate, temperature, development}).$$

If we ignore the last three factors, which can be maintained constant, the function becomes less complicated, but its actual nature is still a matter of uncertainty, although various suggestions as to its form have been made. As early as 1862 Bunsen and Roscoe ⁽²⁾ suggested a relation of the form $S = \log . kJt$, which was modified by Schwarzschild ⁽³⁾ to $S = \log . kJt^p$. Stark ⁽⁴⁾ found that his results agreed with $S = \log . kJ^m t^p$ (in which m and p depend on λ). Hnatek ⁽⁵⁾ holds that k , m , and p are not constant for different kinds of plates, and again Kellner ⁽⁶⁾ finds that the relation between S , J , and t is extremely complicated, and that he can only reconcile his results

by adopting an expression involving five constants. The relation between S and J , which must be intimately connected with the photo-chemical process, is, however, too complex to permit of mathematical expression. As the blackening varies considerably with λ , the comparison of two intensities in different parts of the spectrum must involve a correction for this variation⁽⁷⁾. This can be made through a subsidiary calibration.

Fig. 1.



The general form of the curve representing $S=f(\log J)$, *i.e.*, for t and λ constant, is shown in fig. 1.

The doubt as to the mathematical form of the curve makes it necessary to obtain the characteristic curve by an experimental calibration. Intensity ratios of lines must be deduced from this empirical curve. The characteristic curve of each plate must be obtained independently since the curve varies from plate to plate, even though these be of the same kind and taken from the same box.

Calibration of Plates.

By varying the intensity of the light incident on the plate in known ratios, the exposure time remaining constant, we can produce images of varying densities on the exposed part of the plate. The measurement of these densities by some photometric method gives the data for the correlation of the blackening S and the intensity J^* . By means of this curve we may deduce intermediate values of J from a knowledge of the corresponding S values, and so compare two intensities. As mentioned above, if the two lines have different wave-lengths, then a correction must be applied for the variation of S with λ .

The first type of calibration may be effected by any suitable means whereby the intensity of the incident light can be varied by known amounts. The methods for varying the light intensity fall roughly into two groups: (a) weakening of beam; (b) variation of aperture. The first type may be subdivided into methods involving the use of a neutral wedge which weakens the beam by absorption, the variation being continuous, and the methods by means of which the beam is weakened in steps, *e. g.*, by gauzes, layers of spluttered metal film, etc. The aperture method is based on the principle that, within certain limits, the light passing through a slit is directly proportional to the width of the slit.

Methods of obtaining Calibration Marks.

The wedge method of calibrating plates has been developed and described fully by Merton and Nicholson⁽⁸⁾, and is not considered here. Methods incorporating the use of revolving sectors, etc., involve the "intermittency effect," about which there seems to be considerable divergence of opinion—*e. g.*, Goos⁽⁹⁾ claims that the method gives accurate results, while Kellner⁽¹⁰⁾ found an error of as much as 30 per cent. under certain conditions. Another interesting method employs the variation of intensity of the light source itself. This may be obtained by increasing or decreasing its brightness—*e. g.*, by means of Nicol prisms⁽¹¹⁾, varying the output of source⁽¹²⁾, etc., or by varying its distance from the instrument and calculating, by

* It is customary to plot S against $\log J$, since the curve obtained is more convenient.

means of the "inverse square law," the intensity change⁽¹³⁾.

Variation of Aperture.

A more convenient method, however, employs the use of a variable slit. This principle has been adopted by a number of experimenters, and modified to suit the circumstances of their investigations⁽¹⁴⁾. If the instrument has a good slit, the marks may be obtained by allowing the light from a constant source to be incident on the slit, which has been set at a specified width. The exposure is made for a fixed time, and the plate is then moved to a second position by means of the adjustable plate-holder, and a further exposure made with a different slit-width. This process is repeated with varying slit widths for, say, ten exposures. In this manner ten strips, each corresponding to a known intensity, are produced on the plate. When the density of each image has been measured S and $\log J$ can be correlated. This method proved rather laborious, and had other disadvantages which were overcome by constructing a sliding diaphragm for the front of the slit of the type shown in fig. 2 (Pl. XIV.), which is a contact print of such a diaphragm.

Six slots, 1 mm. wide, were cut in a strip of brass and tin-foil carefully arranged over the spaces so that a series of slit-widths were obtained which, when slipped in front of the spectrograph slit, gave a range of graded apertures. This method had many advantages over the other methods mentioned above (exclusive of the wedge method). A considerable economy was effected in available space on the plate, and the method was independent of variations of intensity in the light source (any variation being common to all the apertures). The apertures were standardized once for all by measurements of their widths on a travelling micrometer, and the calibration marks could be put on at one exposure, thus eliminating any errors due to inaccuracy in measuring the exposure time. The fact that the aperture was uniformly illuminated had to be verified by taking an exposure of the full spectrograph slit with the diaphragm removed. The cross-section of the developed image must give a "uniformly blackened" value when photometered along a direction corresponding to the length of the slit. The lamp used to illuminate the

slit during these experiments was a straight filament lamp specially constructed by the General Electric Company, of Wembley. A large defocussed image was thrown on the slit by a good lens, thereby obtaining uniform illumination. At first a focussed image was used, but difficulties were experienced due to unexpected factors, such as the non-uniformity of the glass of the lamp-bulb. When an opal source was tried, two rings of different intensity were clearly visible. The type of developed image obtained by this method is shown in the upper part of fig. 5 (Pl. XIV.).

Other variations of this method were tried, such as the use of a diaphragm with a wedge-shaped slit (see fig. 3, Pl. XIV); this seems particularly suitable in special cases, and is being developed further.

Weakeners Method.

Since it was desired, in later experiments, to use a 21-ft. Rowland grating with an astigmatic mounting, a method used by Harrison ⁽¹⁵⁾ was tried. The principle, briefly, is that light from a steady source is rendered parallel by a good lens, and a weakening device is introduced into the parallel beam of light so that the image of the source formed by a second lens is weakened in intensity by varying amounts corresponding to the strength of the weakener. The weakeners used were gauzes built on to circular brass frames and constructed so as to fit interchangeably on to a rocking device driven by a constant D.C. motor. It was found necessary to have the gauzes rocked to avoid diffraction effects. Each of the screens was calibrated separately with a linear surface thermophile and micro-potentiometer. During the calibration it was found that the product of the separate weakening effects of the two gauzes was equal to their resultant weakening power when rocked together. The following table shows how accurately this is obeyed.

It was found that the weakening power as measured by the thermopile differed slightly from the value calculated from a knowledge of the area of the effective transmitting region. The general arrangement of the apparatus used in the calibration was the same as that used by Harrison. When operated in conjunction with the grating instrument, the plate was moved in a horizontal direction, a

small part only of the continuous spectrum being allowed to fall on the plate. This was effected by a cover pierced with four slots inserted immediately in front of the plate. The type of exposure is shown in fig. 4 (Pl. XIV.), which is a reproduction of half of a typical plate. This method of varying the intensity of the light source was also employed on the E.1 Spectrograph, and marks obtained by means of it, together with those obtained by the step diaphragm, are shown in fig. 5 (Pl. XIV). These slides afforded a means of comparing the two methods. Calibrated slides of this type were obtained for various standard plates—Wellington Anti-screen, Ilford Special Rapid Panchromatic, Rapid Process Panchromatic, and Iso-Zenith Plates.

TABLE I.

Gauze combination.	Weakening power.	
	Calculated.	Observed.
8×5	0.2048	0.2061
8×7	0.3263	0.3276
6×5	0.1454	0.1433
6×4	0.0573	0.0565

Development.

In each case standard conditions of development were maintained. A standard developer, made up from components measured out by means of a pipette, was freshly prepared for each development. The same developing, fixing, and washing with stigmatic mountings were rigorously observed. A special swirling device was used for developing which consisted of a partition moving backwards and forwards along the dish, thereby promoting an extremely turbulent flow of the developer over the plate. This was found necessary in order to reduce the Eberhard effect⁽¹⁶⁾. In spite of these elaborate precautions, however, it was found that results showed some uncertainty for positions on the plate at distances less than 1 cm. from the edge. This may have been due to variation in the emulsion.

Selection of Method of Calibration.

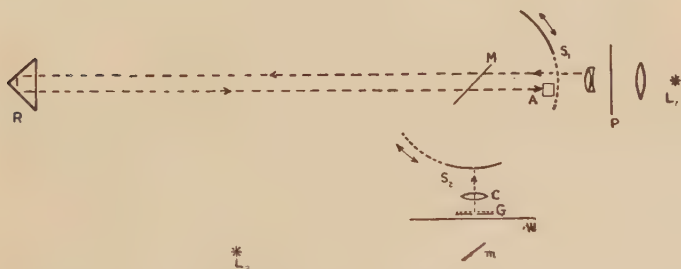
The method of calibration to be adopted in any particular investigation depends to a large extent upon the

spectroscopic apparatus available. In the case of spectrographs of the prism type, or gratings with astigmatic mountings, the wedge method or aperture methods are convenient. With a grating instrument of astigmatic type the astigmatism limits the choice of method, and calibration by a step-weakeners is to be preferred. In some cases the calibration marks may be obtained by transferring the plate from the grating instrument, after the spectrum has been recorded, to a prism instrument, and then using the aperture method. This cannot be used in most cases, however, owing to the properties, such as selective reflexion, absorption, etc., which vary in instruments of the grating and prism type. For accurate work the calibration marks must be obtained under the same conditions as the spectrum.

The Photometer.

The photometer was a modified form of that used by Toy⁽¹⁷⁾ and Childs⁽¹⁸⁾, and is shown diagrammatically in fig. 6.

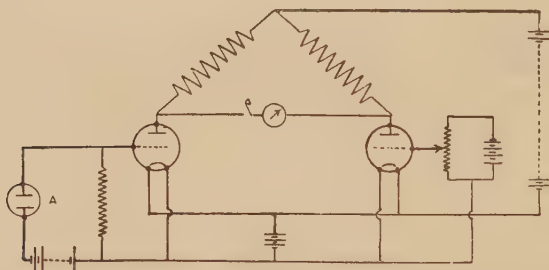
Fig. 6.



Light from a constant lamp L_1 illuminated the plate P under examination. An image of the illuminated part of the plate was then produced on a light-sensitive selenium cell A by reflexion at a right-angled prism R . The illumination from the source L_1 could be cut off by means of a double shutter S_1S_2 , which, on closing at L_1 , opened at S_2 . When the shutter is open at S_2 light is incident on the cell from the ground-glass screen G , in front of which a wedge W is situated. The section of the ground-glass screen is focussed in one plane only by means of a cylindrical lens C . By moving the wedge W the length of the

ground glass in the direction of non-focus is diminished, and so the image on the cell is reduced in intensity only and not in dimensions. By rapidly switching from position S_1 to S_2 , and adjusting W , a point can be reached at which the effects produced on the cell by the image from the plate and from the ground-glass screen are identical. The position of the wedge is then noted. Independent experiments showed that the combination of wedge, ground-glass screen, and cylindrical lens gave a linear "cut-off" of intensity. This is interesting, because it shows that the photometer may be used as an absolute measurer as well as for comparison purposes. The response of the cell to the two sources was compared by a null method, a very sensitive bridge circuit⁽¹⁹⁾ being used (fig. 7). The photo-

Fig. 7.



meter was extremely sensitive and had a high resolving power, the line on the cell being only 0.17 mm. broad and the lens having a magnifying power of about 50.

Results.

Readings were obtained by means of the photometer described above for nine different positions along the plate. The wave-lengths corresponding to these positions were found from a plate calibrated with an iron arc spectrum. Density readings were obtained for both types of calibration marks, the background value being taken as the mean of that on either side of the mark. Usually these two readings were the same. The final value adopted was computed from a large number of random readings, and so the effect of the grain of the plate was eliminated to a large extent. A typical table of $\frac{i_0}{i}$ values is shown in Table II.

TABLE II. $\frac{i_0}{i}$ values.

(a) For Gauze Method of Calibration.

Indicating letter on graph.	Gauzes.										Corresponding wave-length.
	10.	9.	8.	7.	6.	5.	4.	3.	2.	1.	
A	27.4	20.0	13.0	7.66	6.11	3.38	2.96	2.41	1.58	1.07	4865
B	24.7	17.5	10.8	6.38	5.50	3.16	2.77	2.20	1.45	1.15	4985
C	31.0	21.9	13.8	9.05	6.55	3.79	3.10	2.54	1.58	1.20	5115
D	50.0	37.9	22.9	14.6	11.1	5.64	5.22	3.79	2.26	1.41	5240
E	58.6	50.0	35.7	25.4	29.7	19.0	8.10	5.90	2.90	1.60	5390
F	72.3	68.0	51.5	42.5	30.9	16.7	13.6	10.3	4.90	2.40	5550
G	73.3	69.5	62.0	57.1	45.1	32.0	25.3	16.6	9.70	4.20	5770
H	75.0	71.4	68.1	58.8	52.6	48.3	44.1	36.5	21.1	8.20	6220
I	70.0	66.6	57.1	45.9	40.6	32.9	25.5	18.9	9.71	3.52	6800

The top set of readings corresponds to the calibration marks of the gauze method, while the lower set is for the forked aperture. The mean value in each case was computed from at least five independent readings.

(b) For Fork Aperture Method.

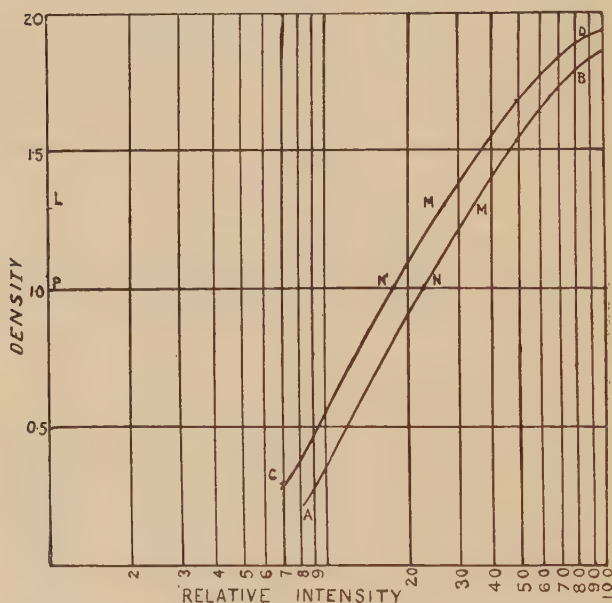
Indicating letter on graph.	Apertures.						Corresponding wave-length.
	6.	5.	4.	3.	2.	1.	
B	28.1	15.6	9.09	4.78	1.55	1.34	4985
F	74.4	64.0	48.5	22.9	5.00	2.50	5550
G	81.6	72.1	59.6	36.5	9.39	4.25	5770

(a) *The Agreement of the Two Methods of Calibration.*

The agreement of the two methods may be shown by plotting the two characteristic curves corresponding to a particular wave-length. The resultant curves are shown in fig. 8. Similar sets obtained from different makes of plates, and corresponding to different wave-lengths, gave like agreement. The two curves are separated because the values of J are on a percentage basis, the actual intensity in one case being greater than in the other. This maximum

intensity is controlled by the extreme aperture, so that if the aperture used for the step-weakeners method is larger than the maximum aperture in the fork, then the absolute value of its intensity will be greater than that from the fork. The intensities are recorded as percentages of this maximum value. From fig. 8 we can see at once that the two methods are in excellent agreement. Consider two arbitrary values of S (the blackening) represented by L and P . Then the corresponding intensities are repre-

Fig. 8.



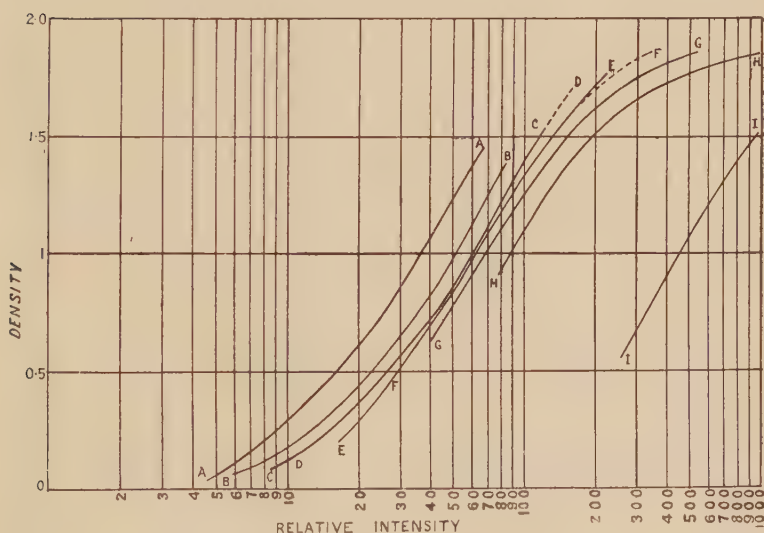
sented by LM , LM' , and PN , PN' . If the two methods agree, then the ratio of the corresponding intensities, as deduced from these curves, must be equal, *i. e.*, $\frac{LM}{PN} = \frac{LM'}{PN'}$. When the measured values are substituted in these ratios, as seen from the graph, they are identical.

(b) *Variation of Density with λ .*

Again, if we know the energy distribution of the light emitted by the lamp used for calibrating the plates, we

can determine from the tables of results the selective response of each of the plates for the particular range of λ under consideration. The special G.E.C. lamp was calibrated by means of a Hilger Quartz Monochromator and a thermopile attachment used in conjunction with a Paschen Galvanometer, which was only steady enough for measurement between midnight and 3 A.M. The dispersion variation and absorption and reflexion losses were allowed for⁽²⁰⁾, and an energy wave-length curve ($E_\lambda : d\lambda$) obtained for the lamp. The dispersion curve for the E.1

Fig. 9.



was also obtained, and, taking into account the dispersion variation, a curve representing the relative amounts of energy falling on the plate for different wave-lengths was ascertained. In plotting the density curves a uniform scale was preserved by multiplying the intensities by factors deduced from this latter energy curve.

The plates chosen—Ilford Special Rapid Panchromatic, Rapid Panchromatic, Iso-Zenith, and Wellington Anti-Screen—were selected because they are typical of those used in spectral photography, giving a range of different

speeds, contrasts, etc.* The respective characteristic curves which we have obtained are shown below in figs. 9, 10, 11, 12.

Fig. 10.

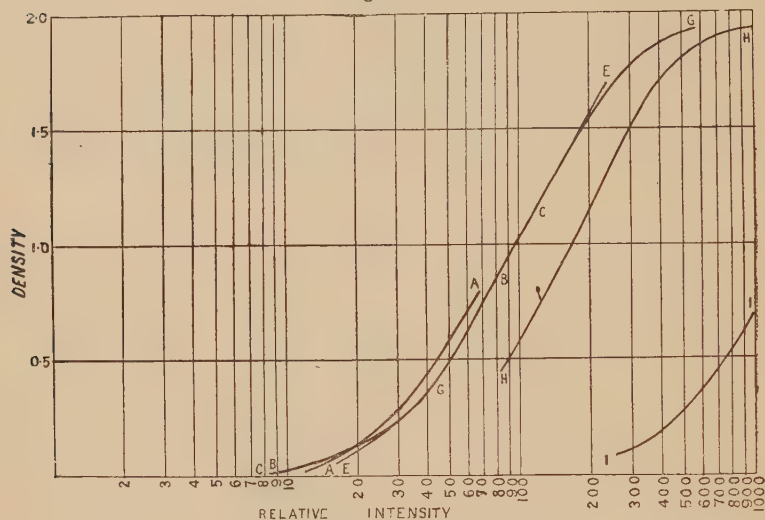
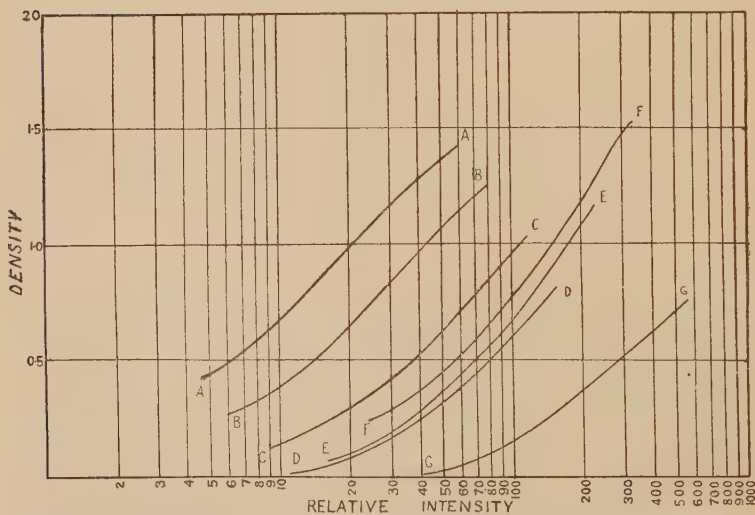


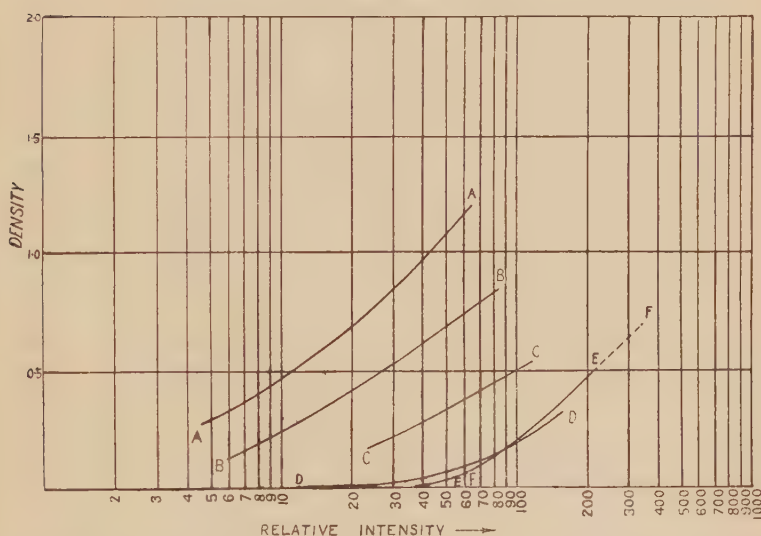
Fig. 11.



* Similar investigations have been made by Sandvik and Silberstein (21) on Eastman Kodak plates.

The curves obtained by the above procedure vary from those which might be anticipated by visual observation of the variation in blackness. For example, the Special Rapid plates show a minimum density just visible at the left of fig. 5 (Pl. XIV.), which is followed by a second maximum. (As only half the plate is reproduced this is not shown.) This second maximum is due to the fact that the plate is very sensitive to this wave-length, the blue, and although the energy from the lamp in this region is small, the increased sensitivity more than counterbalances the deficiency, and so gives the more dense image. The

Fig. 12.

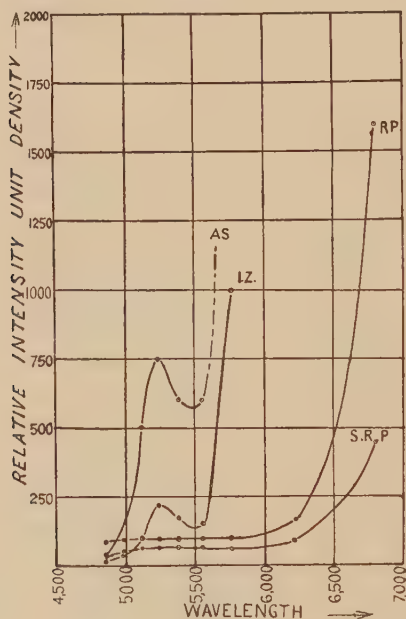


maximum to the right of fig. 5 (Pl. XIV.) is due to the large amount of energy in the source for this—the red—region.

The curves given above indicate the type of response to be expected from the different plates. This can be more conveniently represented by considering some fixed value of density and measuring the corresponding intensities required to produce this density. Fig. 13 represents the variation graphically. The common density chosen was unity, since in the majority of cases it is on the straight part of the characteristic curve. The curve is, in a way, a measure of the relative sensitivities of the plates, and

from its variation with type can at once be seen. Had some different value of density been chosen for this comparative graph, the curves would have had a slightly modified shape, but would have been of the same form. The curves of variation of contrast, λ , with wave-length are shown in fig. 14.

Fig. 13.



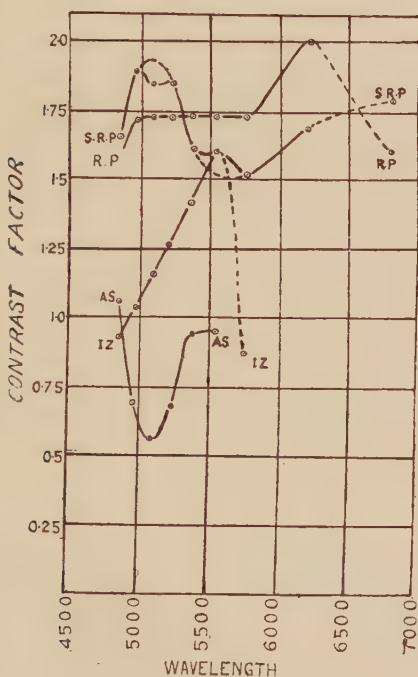
Choice of Plates for Spectroscopic Work.

The nature of the characteristic curve which is the chief controlling factor in the choice of a photographic plate for any particular photometric investigation is shown in fig. 1. It may be divided, for convenience of discussion, into three sections—an upper curved part BD and a lower curved part AC, known respectively as the “over”- and “under”-exposed regions, and a straight portion AB. The central straight portion can be represented by an equation of the form

$$S = \gamma \cdot \log J - A_0.$$

The constant γ , which is the gradient of the straight-line portion, is known as the "contrast" factor of the plate. The greater its value the larger the difference in density produced for a fixed difference in intensity. A high value of γ is not necessarily an advantage from a photometric point of view. The best value for measuring purposes is that corresponding to $\gamma = \tan 45^\circ$, the usual value of

Fig. 14.



slope for most accurate measurement in experimental observations. It is an advantage to have the length of AB as large as possible, since larger differences in intensity may then be measured.

The value of A_0 is given by the intercept of the straight portion on the "log J" axis, and is known as the inertia of the plate. If the curve had no "heel," and continued straight down to the axis at P, then the inertia is represented by OP. The speed of the plate is usually taken to

be proportional to the reciprocal of the "inertia." The speed as generally given on a box of plates is expressed in the form, say, H. & D. 700. This H. & D. (Hurter and Driffield) number, obtained from the ratio $34/(\text{inertia})$ for development under certain standard conditions, is expressed in foot-candle-seconds. Plates are designated slow, medium, or fast, according to their inertia values.

In general these speed ratios are for white light, and, from a spectroscopic point of view, are only approximate. The speed depends to a considerable extent on the wave-length of the light being measured. As a general rule fast plates have low contrast, and *vice versa*.

A further factor has to be taken into account in the choice of plate for spectroscopic work, viz., the "grain" size. High-speed plates have large grain size; consequently the developed image is not so uniform, and the edge is not so sharp as in the case of medium or, even more so, slow plates. In addition to this decrease in resolution with high-speed plates, there is a tendency to "fog," the background being darkened on development as if the whole plate had been subject to a faint illumination. Increase in the background density decreases the available photometric range, and so is disadvantageous.

When high resolution is desired slow plates must be used. These have high contrast, and weak lines, together with the "fog" effect, are eliminated.

Conclusion.

The many factors enumerated above would appear to make the problem of spectral photometry *via* photographic process very complex. No assumptions may be made with any degree of certainty as to the behaviour of a particular plate, and consequently a characteristic curve must be obtained for each plate, and also, if the variation with wave-length of the lines is large, for each wave-length of the lines under examination. When this procedure is adopted, and there seems to be no alternative, the photographic recording of a spectrum, together with the calibration marks, affords a method for photographic measurement which is capable of moderate accuracy. Many of the difficulties are eliminated by individual calibration.

If the plates are chosen judiciously, with due consideration for the factors mentioned above, and if the mean value

of a large number of readings is taken, then relative intensities may be deduced to an accuracy of about 5 per cent.

We should like to take this opportunity of expressing our deep gratitude to Dr. R. C. Johnson for continuous help and valuable criticism throughout the progress of this work. We should also like to thank Mr. C. W. Oatley, M.A., M.Sc., for valuable help and advice in connexion with the valve amplifier employed in the photometer.

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XCIX. *Changes in the Dimensions of Metallic Wires produced by Torsion.*—II. *Silver, Gold, Aluminium, and Nickel.*
By THOMAS LONSDALE, M.Sc., Ph.D. (London), F.Inst.P.
(British Silk Research Association)*.

IN Part I. ⁽¹⁾ of this paper the changes in length were described which occur when high-conductivity soft drawn copper wire under various tensions is twisted at

* Communicated by the Author.

room temperature until the breaking point is reached, each tension applied, when acting alone, being too small to produce elongations measurable with the apparatus employed.

It was found that if e was the elongation of the wire as a percentage of the initial length, T the twist put in, in turns per cm., t the initial tension in kg. per sq. cm., D the diameter in cm., then the expression $e = 4.9 DT \times (1 + 0.0065t)$ roughly gives the relationship existing within the limits of 0–1.05 for DT , and 10–500 for t , for values of D of 0.071, 0.037, 0.023, and 0.012 cm. This expression is better written $e = DT(a + bt)$, where a is 4.9 and b is 0.032.

The wire occasionally breaks when an elongation of 3–4 per cent. is reached, but elongations of 10–12 per cent. have been observed.

The present paper gives the results of similar experiments made on silver, gold, aluminium, and nickel, and completes the study of the commoner metals of the face-centred cubic class with the exception of lead, which shows peculiarities to be discussed in Part III.

Copper, silver, and gold belong to the same group in the periodic classification and are physically and chemically somewhat similar in properties; aluminium and nickel belong to different groups. The experiments described in this paper show that the effect here discussed, for metals of the face-centred cubic class, depends upon the hardness of the metal rather than upon its position in the periodic classification.

The apparatus used was that described in Part I. ⁽¹⁾

Details regarding Metals Studied.

Silver.—Silver was studied in detail. Experiments were made on 22, 28, 34, and 40 S.W.G. wire prepared by Messrs. Johnson Matthey as follows:—A small ingot of assay silver which was spectroscopically almost pure was drawn down to 22 S.W.G. and then annealed; the required quantity of 22 S.W.G. wire was then cut off, and the remainder drawn down to 28 S.W.G. and annealed; this procedure was repeated for each of the four sizes of wire required. Messrs. Johnson Matthey do not wish to make public their method of annealing, except to say that each anneal was very thorough and according to the same standardized procedure.

Gold.—The wire was assay gold, 34 S.W.G., spectroscopically almost pure, drawn and subsequently annealed by Messrs. Johnson Matthey & Co.

Aluminium.—The wires were commercially pure aluminium of 28, 30, and 36 S.W.G., part of samples kindly given to me by the British Aluminium Company. All the material was drawn down from the same coil of hot rolled rod having the following composition :—Si 0.13 per cent., Fe 0.21 per cent., Cu trace, Al 99.66 per cent., and was then wound on aluminium tubes, annealed for 2 hours at 430–440° C., and sent to me on the aluminium tubes, to avoid the hardening produced by rewinding on to wooden bobbins.

Nickel.—The wires were 28, 33, and 38 S.W.G., commercially pure nickel specially drawn for me by Messrs. Henry Wiggin & Co. The following is a typical analysis of the material used :—Cu 0.08 per cent., Fe 0.10 per cent., C 0.08 per cent., Si 0.20 per cent., Ni 99.50 per cent. The three gauges of wire were annealed separately in a neutral or reducing atmosphere for about 30 minutes, the 28 and 33 S.W.G. at 700° C. and the 38 S.W.G. at 650° C., after which each set of coils was rewound, and the coils were annealed together at 725° C. for 45 minutes in air.

Rate of Twisting.

In Part I. it was found that for soft drawn copper the elongation produced by a given twist and tension was independent of the rate of twisting within the range of rates of twisting it was convenient to employ, so that in the case of soft drawn copper no particular attention was paid to regulating the rate of twisting. In a few preliminary experiments this independence was shown also for silver, gold, aluminium, and nickel, within the range of rates of twist used (2–14 turns per metre per sec.), but the behaviour of lead was found to depend in a complex manner on the rate of putting in the twist. It was therefore thought well to adopt a standardized method of twisting. Preliminary experiments showed that for each metal the elongation for a given twist and tension was roughly proportional to the diameter of the wire; the rate of twisting for each metal was arranged to be roughly inversely proportional to the diameter of the wire, so that the rates of elongation of the wires of each metal of different diameters, due to the torsion, were roughly the

same under the same tension. The following table gives the actual rates of twisting employed:—

TABLE I.

Rates of Twisting Wires of Silver, Gold, Aluminium, and Nickel.

Metal.	S.W.G.	Initial diam. (cm.)	Rate of twisting (turns per metre per sec.).
Silver	22	0.0708	2/3
	28	0.0380	4/3
	34	0.0227	2
	40	0.0120	4
Gold.....	34	0.0227	2
Aluminium.....	28	0.0378	4/3
	30	0.0316	5/3
	36	0.0194	8/3
Nickel.....	28	0.0378	4/3
	33	0.0248	2
	38	0.0149	10/3

In making the experiments increments of twist were put in by hand, turning against a metronome, for the first 30 seconds of each consecutive minute, the corresponding elongations being noted during the last 30 seconds.

The initial length of wire under test in each experiment was 1 metre, except in the case of gold, where, to economize material, the initial length was 50 cm.

Tension.

The maximum initial tension on the wires was chosen after preliminary experiments to be such that, acting alone in the absence of torsion, it produced a permanent elongation of not more than 0.02 per cent. in 24 hours. This tension was 500 kg. per sq. cm. for silver, 250 kg. per sq. cm. for gold and aluminium, and 1000 kg. per sq. cm. for nickel. It was shown by further experiments that when these loads were left on wires after they had been highly twisted the elongations produced were not

sensibly greater than those given above, in spite of the fact that the corresponding tensions were increased on account of the decrease in diameter associated with the elongation due to twisting.

Discussion of the Results.

Some of the results obtained are shown graphically in figs. 1-3. In the case of silver, aluminium, and nickel, where results on two or more diameters are given, the scales of twist are made roughly proportional to the diameters of the wires. This device was previously employed for the results on soft drawn copper (fig. 2) ⁽¹⁾.

Effect of Diameter.

For silver, as in the case of copper, the diameters of the four gauges of wire studied are in the ratio of approximately 6 : 3 : 2 : 1. In the curves for silver in fig. 1 the scales of twist have been adjusted to be in the same ratio—that is, the scale of the first set is twice that of the second, three times that of the third, and six times that of the fourth. It will be seen that, as in the case of copper (Part I.), the four sets of curves occupy approximately the same space, showing that the elongation for a given twist and tension is roughly proportional to the diameter of the wire. Similar adjustments of the scales of twist have been made for the less extensive results for aluminium and nickel (fig. 3); these results and other results not now given for lead show the same relationship for wires of the same metal of different diameters.

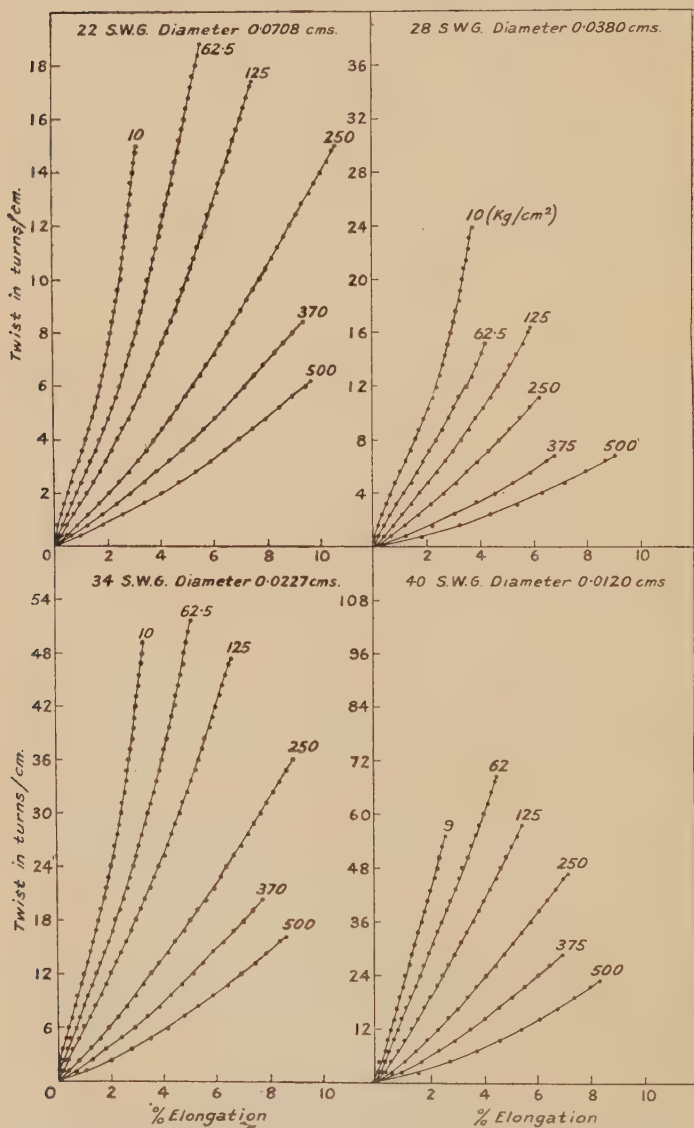
These results would suggest that for metal wires of the face-centred cubic class in the annealed state the elongation e is a function of the twist T , the diameter D , and the tension t of the form $e = DTF(t)$, where $F(t)$ involves constants characteristic of each metal in the class. (This relationship has been studied only for copper, silver, aluminium, nickel, and lead, for wires of diameter not greater than 22 S.W.G., except in the case of lead.)

Effect of Tension.

As the wire elongates its diameter decreases slightly, and the tension corresponding to a given load therefore increases. This increase, however, is only a few parts

Fig. 1.

Twist-extension Curves for Soft Drawn Silver Wire of various Diameters under various Tensions.

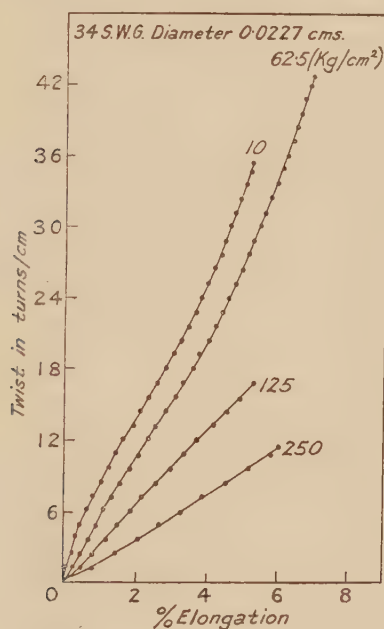


(The number at the top of each curve is the initial tension on the wire in kg./sq. cm.)

per cent., and has been neglected. Neglecting the sinuosity of the curves, it may be stated empirically that, at room temperature, for all the metals studied (*i. e.*, copper, silver, gold, aluminium, nickel, and, under a standard rate of twisting, lead) the percentage elongation varies approximately as $a + bt$, where a and b are constants

Fig. 2.

Twist-extension Curves for Soft Drawn Gold Wire of various Diameters under various Tensions.



(The number at the top of each curve is the initial tension on the wire in kg./cm.².)

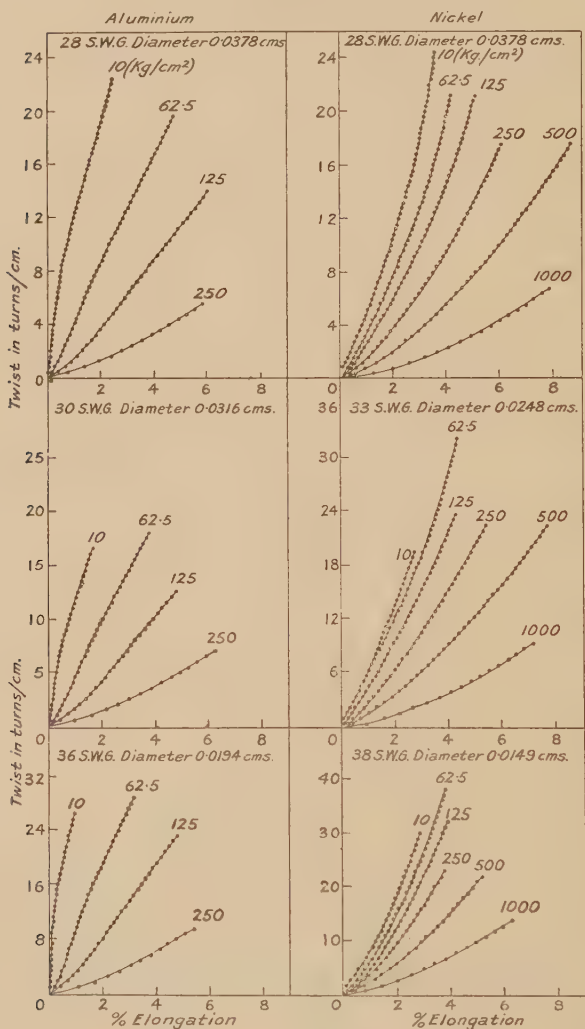
for the particular metal; b is always positive, a may be positive or zero.

Effect of Twist.

As with copper, so with all the metals studied, the elongation produced by the combined action of tension and torsion is enormously greater than that produced

Fig. 3.

Twist-extension Curves for Annealed Wires of various Diameters under various Tensions.



(The number at the top of each curve is the initial tension on the wire in kg./sq. cm.)

by the action of tension alone ; for a given diameter and applied tension the percentage elongation increases almost directly with the amount of twist.

Detailed Discussion of the Results.

Silver.—Silver was studied in some detail, since in its assay form it is obtainable relatively easily in an exceedingly high state of purity. To give a direct comparison with copper (Part I.) the same sizes of wire were used in both metals.

The curves in fig. 1 show some of the results obtained. As might be expected, the curves are qualitatively very similar to those given for soft drawn copper, but for low tensions the curves are rather less S-shaped than the corresponding curves for copper. For low tensions the elongations are less than the corresponding elongations for copper, but the influence of the tension on the elongation is rather greater for silver than for copper, so that for higher tensions the results for silver are quantitatively similar to those for copper. For the higher tensions employed the lower part of the S has disappeared and the rate of extension per unit twist is greatest at the origin.

The experimental results obtained can be represented approximately by the expression $e=DT(a+bt)$, where $a=3.4$ and $b=0.045$. The fit of this expression to the experimental results is shown in Table II., which gives experimental values of e and corresponding calculated values.

This expression has been studied within the limits of 0–1.05 for DT and 10–500 kg. per sq. cm. for t for values of D of 0.071, 0.038, 0.023, and 0.012 cm.; the wires are apt to break for values of e greater than 2–4 per cent., though elongations of 8–10 per cent. have been obtained.

A study of the curves in fig. 3 and the figures in Table II. reveals peculiarities in the case of the 28 S.W.G. wire; the elongations for a given twist and tension are greater than might be expected from the results on the other gauges of wire, and the wires themselves break sooner; it would appear that these peculiarities must be due in some way to the drawing of this gauge of wire rather than to the annealing of the wire or to impurities, since the wire is almost spectroscopically pure and the annealing was carried out according to a standardized procedure.

When considerable twist has been put in, for the tension of 10 kg. per sq. cm. the wire shows a tendency to assume a spiral form, the length between the two clamps being

TABLE II.

Experimental Values of the elongation for Silver and corresponding calculated Values, using the Expression $e = DT(3.4 + 0.045t)$.

D=diameter of the wire in cm.

L=load in gm.

t =initial tension in kg. per sq. cm.

T=twist in turns per cm. of the initial length.

e' =experimental value.

e =calculated value of elongation, per cent. of the initial length.

D=0.0708.						22 S.W.G.						
L	...	39	...	246	...	492	...	985	...	1477	...	1969
<i>t</i>	...	10	...	62.5	...	125	...	250	...	375	...	500
DT.	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>
0.15	0.6	0.6	0.9	0.9	1.4	1.3	2.2	2.2	3.0	3.0	3.9	4.3
0.30	1.2	1.3	1.9	1.7	2.7	2.5	4.4	4.0	6.1	5.5	7.8	7.4
0.45	1.7	1.9	2.8	2.6	4.1	3.6	6.6	5.5	9.1	7.5		
0.60	2.3	2.3	3.7	3.2	5.4	4.4	8.8	6.9				
0.75	2.9	2.6	4.6	3.8	6.8	5.2	10.9	8.1				
0.90	3.5	2.9	5.6	4.2	8.1	6.0	13.1	9.3				
1.05	4.0	3.1	6.5	4.7	9.5	6.7	15.3	10.5				
1.20	4.6	3.4	7.4	5.2	10.8	7.4						
1.35	5.2	3.7										
D=0.0380.						28 S.W.G.						
L	...	12	...	71	...	142	...	283	...	425	...	566
<i>t</i>	...	11	...	62.5	...	125	...	250	...	375	...	500
DT.	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>
0.15	0.6	0.7	0.9	1.1	1.4	1.7	2.2	2.6	3.0	4.5	3.9	6.2
0.30	1.2	1.6	1.9	2.3	2.7	3.3	4.4	4.7				
0.45	1.8	2.4	2.8	3.5	4.1	4.6						
0.60	2.3	3.0			5.4	5.8						
0.75	2.9	3.4										
D=0.0227.						34 S.W.G.						
L	...	4	...	25	...	50	...	99	...	149	...	198
<i>t</i>	...	10	...	62.5	...	125	...	250	...	375	...	500
DT.	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>	<i>e.</i>	<i>e'.</i>
0.15	0.6	0.5	0.9	0.7	1.4	1.1	2.2	2.1	3.0	3.1	3.9	4.4
0.30	1.2	1.1	1.9	1.6	2.7	2.2	4.4	3.9	6.1	5.5	7.8	7.3
0.45	1.7	1.7	2.8	2.4	4.1	3.3	6.6	5.5	9.1	7.6		
0.60	2.3	2.2	3.7	3.1	5.4	4.2	8.8	6.9				
0.75	2.9	2.6	4.6	3.7	6.8	5.0	10.9	8.3				
0.90	3.5	2.9	5.6	4.2	8.1	5.7						
1.05	4.0	3.2	6.5	4.7	9.5	6.4						

TABLE II. (*con.*).

D=0.0120. 40 S.W.G.												
L	...	1	...	7	...	14	...	28	...	42	...	57
t	...	9	...	62	...	125	...	250	...	375	...	500
DT.	e.	e'.		e.	e'.		e.	e'.		e.	e'.	
0.15	0.6	0.6		0.9	0.8		1.4	1.3		2.2	2.4	3.0 3.6 3.9 6.3
0.30	1.1	1.2		1.9	1.7		2.7	2.6		4.4	4.3	6.1 6.2
0.45	1.7	1.8		2.8	2.6		4.1	3.8		6.6	6.0	
0.60	2.3	2.4		3.7	3.4		5.4	4.9				
0.75				4.6	4.1		6.8	6.0				

0.1–0.2 cm. less than the actual length of the wire. In order to measure the true length of the wire it was straightened by giving to it an additional tension of 125 kg. per sq. cm. while a measurement was being made; this was removed again before the next increment of twist was put in.

Gold.—It was not thought necessary to make a study of the effect of diameter in the case of gold, since the results on the other metals show fairly clearly the nature of the relationship to be expected; experiments were made only on wire of diameter 0.0227 cm. (34 S.W.G.).

The wire was assay gold, spectroscopically almost pure.

The extensions produced by twisting are much greater for gold than for silver or copper; some of the results obtained are shown graphically in fig. 2; in general features these curves are more like those given by copper than those given by silver; for the lowest tension gold gives a curve which is markedly S-shaped; in this it resembles copper more than silver.

Neglecting the sinuosity of the curves, and assuming that the diameter relationship found in the case of the other metals holds also in the case of gold, then the relation between twist and tension can be represented approximately by the expression $e = DT$ ($3.3 + 0.083 t$). Table III. shows experimental values of e and corresponding calculated values using this expression.

A study of Table III. shows that the fit of the expression to the experimental results is fairly good except for the higher torsions for the lowest load, 10 kg. per sq. cm.

The gold showed no tendency to curl even under the lowest tension; no auxiliary tension was therefore applied when reading the elongation.

Aluminium.—Some of the results obtained are shown graphically in fig. 3. All the twist-extension curves for aluminium are slightly concave towards the twist axis in the neighbourhood of the origin. For the lowest tension (10 kg. per sq. cm.) and greater twist the curves are convex towards the twist axis; this convexity is less marked for the next higher tension employed, and for still higher tensions the curves are entirely concave towards the twist axis.

TABLE III.

Experimental Values of the elongation for Gold and corresponding calculated Values using the Expression $e=DT(3\cdot3+0\cdot083\ t)$.

(Notation as in Table II.)

D=0.0227 cm.									
L	...	4	...	25	...	50	...	99	
<i>t</i>	...	10	...	62.5	...	125	...	250	
DT.	<i>e.</i> <i>e'</i> .		<i>e.</i> <i>e'</i> .		<i>e.</i> <i>e'</i> .		<i>e.</i> <i>e'</i> .		
0.10	0.4	0.4	0.9	0.8	1.4	1.4	2.4	2.5	
0.20	0.8	1.1	1.7	1.7	2.7	2.7	4.8	4.8	
0.30	1.2	1.9	2.6	2.6	4.1	4.2			
0.40	1.6	2.8	3.4	3.5					
0.50	2.1	3.6	4.3	4.3					
0.60	2.5	4.2	5.1	5.0					
0.70	2.9	4.8	6.0	5.6					
0.80	3.3	5.3	6.8	6.2					
0.90			7.7	6.7					

Among the metals studied the effect of tension on the elongation produced by a given twist was most pronounced with aluminium. Thus under the lowest tension the extensions of the aluminium wire were less than the corresponding extensions for silver, gold, etc., but under the highest tensions they were greater. For a given twist the extension of the aluminium wires (except the two larger diameters twisted under the lowest tension) is roughly proportional to the tension, and the general expression $e=DT(a+bt)$ therefore reduces to the form $e=DT(bt)$.

Table IV. shows the value of the constant b obtained by substituting representative experimental values of e , t , and T ; the mean value of b is 0.11, omitting the values of b for the lowest tension (10 kg. per sq. cm.).

Nickel.—Some of the results obtained are shown graphically in fig. 3. The curves in their general features are very similar to those given by silver.

TABLE IV.

Experimental Values of the Constant b for Aluminium in the Expression $e=DT(bt)$.

(Notation as in Table II.)

D=0.0378 cm. 28 S.W.G.								
L.								
t	...	10	...	62.5	...	125	...	250
DT.	...	b	b	b	b .
0.10		0.16		0.12		0.13		0.14
0.20		0.16		0.10		0.11		
0.30		0.18		0.08		0.10		
0.40		0.22		0.10		0.09		
0.50		0.25		0.10		0.09		
0.60		0.26		0.10				
0.70		0.28		0.10				
0.80		0.29						

D=0.0316 cm. 30 S.W.G.								
L.								
t	...	10	...	62.5	...	125	...	250
DT.		b .		b .		b .		b .
0.10		0.20		0.13		0.13		0.14
0.20		0.20		0.11		0.11		0.12
0.30		0.25		0.10		0.10		
0.40		0.29		0.10				
0.50		0.31		0.11				

D=0.0194 cm. 36 S.W.G.								
I.								
L.								
t	...	10	...	62.5	...	125	...	250
DT.		b .		b .		b .		b .
0.10		0.10		0.11		0.12		0.14
0.20		0.08		0.09		0.10		
0.30		0.11		0.09		0.09		
0.40		0.15		0.09		0.09		
0.50		0.18		0.09				

The elongations of the nickel wires for a given twist are, however, less dependent on the tension than those of the silver wires, or, indeed, than those of the wires of the other metals studied. Thus the elongations of the nickel wires are slightly less under the lower tensions and considerably less under the higher tensions than the corre-

sponding elongations of the silver wires; the contrast between nickel and aluminium is even more marked.

The results are fairly well fitted by the general expression $e = DT(a + bt)$, where b is 0.023. Table V. shows the value of the constant a obtained by substituting representative experimental values of e , t , and T ; the mean value of a is 5.8.

TABLE V.

Experimental Values of the Constant a for Nickel in the Expression $e = DT(a + 0.023 t)$.

(Notation as in Table II.)

D=0.0378 cm. 28 S.W.G.												
L.												
<i>t</i>	...	10	...	62.5	...	125	...	250	...	500	...	1000
DT.		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>
0.15		5.3		6.3		6.4		7.9		9.6		14.8
0.30		5.1		5.4		5.6		5.7		5.4		
0.45		4.7		5.0		4.7		4.6		3.3		
0.60		4.4		4.5		4.0		3.7		2.3*		
0.75		4.0		4.0		3.6						
0.90		3.7										
D=0.0248 cm. 33 S.W.G.												
L.												
<i>t</i>	...	10	...	62.5	...	125	...	250	...	500	...	1000
DT.		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>
0.15		5.8		5.3		6.2		7.0		8.0		12.2
0.30		5.7		5.2		5.4		5.3		4.8		
0.45		5.4		5.0		4.8		4.4		3.2		
0.60		5.1		4.6								
0.75		4.7		4.2								
D=0.0149 cm. 38 S.W.G.												
L.												
<i>t</i>	...	10	...	62.5	...	125	...	250	...	500	...	1000
DT.		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>		<i>a.</i>
0.15		7.8		8.2		8.0		7.1		8.0		10.0
0.30		6.9		6.8		6.4		5.3		4.6		
0.45				5.9		5.4						

The a term is less important for the higher tensions, and therefore the mean expression fits the experimental results for these tensions better than might have been expected from the individual values of a given in Table V. Thus, even for the result marked with an asterisk (*) the calculated extension, using the mean value of a (5.8), is 10.4 per cent., and the observed elongation is 8.1 per cent.

*General Comparison of the Metals Copper, Silver, Gold,
Aluminium, and Nickel.*

The twist-elongation curves for all the metals studied have some approximation to straight lines.

For high tensions the curves for all the metals except gold are concave towards the twist axis; the curve for gold is a straight line. For low tensions the curves for copper, silver, gold, and nickel are slightly S-shaped, the sinusoidal nature of the curves being less marked in nickel than in the other metals; near the origin the curves

TABLE VI.

Values of a and b in the Expression $e=DT(a+bt)$ for various Metals studied, together with Values of the Brinell Ball Hardness as given by Edwards.

(Notation as in Table II.)

Metal.	a .	b .	Brinell hardness (Edwards).
Nickel.....	5.8	0.023	144
Copper.....	4.9	0.032	53
Silver	3.4	0.045	37
Gold.....	3.3	0.083	33
Aluminium.....	0	0.11	25
Lead.....	0	0.5	7

are slightly concave towards the twist axis, but they are subsequently rather more markedly convex towards the twist axis; for low tensions the curves for aluminium are mainly convex towards the twist axis.

In all the metals considerable extensions are produced by the torsion. In aluminium these extensions are comparatively small for very small tensions.

The results for all the metals studied can be represented approximately by the expression $e=DT(a+bt)$, where e is the elongation as a percentage of the initial length, D the diameter in cm., T the twist in turns per cm. of the initial length, t the tension in kg. per sq. cm., and a and b are constants with characteristic values for each metal.

The values of a and b for each metal are collected in Table VI., together with the Brinell ball hardness of each

metal in the annealed state as determined by Edwards ⁽²⁾. Values of these constants are given for lead also, though these give no more than a rough indication of the general behaviour of lead, since the elongation in the case of lead for a given twist and tension is markedly dependent on the rate at which the twist has been put in to the wire. (This will be discussed in Part III.)*

In the expression $e = DT(a + bt)$ the influence of the constant a predominates for low tensions and that of b for high tensions. The value of a decreases in an irregular manner as the Brinell ball hardness decreases; nickel, copper, silver, and gold fall fairly sharply into one group with a value of a of 5.8–3.3, and aluminium and lead into another with a value of a of zero; the elongations of the metals in the second group for very low tensions are comparatively very small.

The value of b increases fairly uniformly as the hardness decreases, except that for a small diminution in hardness from silver to gold there is a large increase in b ; the softer the metal the greater is the value of b and the greater is the dependence of the elongation upon the tension for a given twist.

All the metals studied belong to the face-centred cubic class, so that the general mechanisms of distortion-slipping on crystal planes etc.—both in the case of Brinell test and in the experiments under discussion, are probably the same for all these metals, though different in the two cases (with the possible exception of lead, which may have peculiar properties due to its self-annealing at room temperatures).

For crystals of this class the elongation of a metal wire under the combined action of tension and torsion appears to have some fairly definite relation to those properties of a metal termed “hardness.”

Accuracy of the Results.

Repetition of some of the measurements showed with silver and nickel that the elongations of different portions of wire from the same reel under the same tension for a given twist did not vary by more than one part in twenty, the variation being usually much less than this; the variations for gold and aluminium were much greater than this, differences as great as one part in ten being

* P. 1887, *infra*.

found. These variations indicate a variation in properties of the wire from place to place along its length: this variation is almost certainly of a mechanical nature rather than a variation in composition, since the gold, silver, and probably the lead are exceedingly pure. Experiments made on the distribution of twist in the wires also indicate that such variations in properties are present.

Distribution of Twist in the Wires.

When a length of wire is examined under the microscope fine markings parallel to the axis and of different widths are usually seen on its surface; when the wire is twisted these markings form spirals which sometimes have a very regular appearance. It was found that the number of these spirals on the surface of the wire is much greater than the number of turns of twist put into the wire, so that an examination of these spirals gives no very reliable indication of the regularity of distribution of twist in the wire.

This question of the uniformity of distribution of the twist was studied further in silver, gold, aluminium, and nickel by tying a tiny wisp of thread to the mid point of the wire in some of the experiments. The number of rotations of this thread gives the number of turns put into the upper half of the wire. The difference between this number and the number of turns put into the whole wire (read off on the revolution counter) is the number of turns put into the lower half of the wire.

A few results typical of those obtained are given in Table VII.

The first few turns of twist in the wires distribute themselves somewhat irregularly; the subsequent twist goes fairly uniformly into all the wires. The ratio T_a/T_b is given in the last column of the table; it is some indication of the regularity of distribution of twist in the wires: the greater the departure of this ratio from unity the more irregular is the distribution of the twist.

The departures from unity of the ratio T_a/T_b show that there are slight variations in mechanical properties from place to place along the wires. In view of this, and in view of the irregular departure of the results from any systematic algebraic expression, it has not been thought

justifiable to put forward anything more than the simple and approximate expressions already discussed.

TABLE VII.

Distribution of Twist in Silver, Gold, Aluminium, and Nickel Wires.

T_a =twist in top half of wire.

T_b =twist in bottom half of wire. (turns per cm. of initial length).

(Notation as in Table I.)

Metal.	D.	t .	e' .	T_a .	T_b .	$\frac{T_a}{T_b}$.
Ag ...	0.0380	62.5	2.40	8.10	7.90	1.03
		62.5	3.92	14.36	13.64	1.05
		125	3.34	7.72	8.28	0.93
		125	5.86	15.38	16.62	0.93
	0.0708	62.5	2.78	7.12	6.88	1.03
			5.42	18.14	17.86	1.02
		125	4.26	7.90	8.10	0.98
			7.06	15.88	16.12	0.99
Au ...	0.0227	62.5	4.10	20.56	21.44	0.96
			6.94	40.44	43.56	0.93
		250	3.30	5.84	6.16	0.95
		250	2.86	6.60	5.40	1.22
Al ...	0.0378	62.5	2.94	5.92	6.08	0.97
			6.06	13.74	14.26	0.96
Ni ...	0.0248	62.5	2.44	14.78	15.22	0.97
			4.18	29.26	30.74	0.95
		125	2.46	12.26	11.74	1.04
			4.32	25.08	22.92	1.10

Summary.

The paper gives measurements of the changes in length of the commoner metals of the face-centred cubic class—silver, gold, nickel, and aluminium—in the form of well annealed wire; these changes in length occur while the wire is being twisted at room temperature under a constant load which is small in comparison with the load necessary to produce permanent set. Considerable elongations are produced by twisting even under very small tensions.

If e is the elongation as a percentage of the initial length, T the twist put in in turns per cm., t the initial tension in kg. per sq. cm., D the diameter in cm., then the expression $e = DT(a + bt)$ roughly gives the relationship existing, where a and b are constants having characteristic values for each metal. Arranging the above metals together with lead and copper in the order of decreasing Brinell hardness, the numerical value of a decreases and that of b increases from metal to metal.

My thanks are due to Messrs. Johnson Matthey & Co., Messrs. H. Wiggin & Co., and the British Aluminium Co. for taking great trouble in preparing small quantities of material to my special requirements.

References.

- (1) Lonsdale, *Phil. Mag.* (7) viii. p. 703 (1929).
- (2) Edwards, *Journ. Inst. Metals* xx. no. 2, p. 87 (1918).

C. Changes in the Dimensions of Metallic Wires produced by Torsion—III. Lead. By THOMAS LONSDALE, M.Sc., Ph.D. (London), F.Inst.P. (British Silk Research Association).*

AS already indicated in Part II. of this series of papers on the twisting of metallic wires, lead wires behave somewhat differently from copper, silver, gold, nickel, or aluminium wires. Experiments on the behaviour of lead wires are now described in this present paper.

Material.

The material used was extruded lead wire of diameters 0.293 cm., 0.157 cm., and 0.075 cm., supplied by Messrs. Johnson Matthey as chemically pure lead (probably 99.98 + per cent. lead⁽¹⁾).

Annealing.

It is not customary to anneal lead. It is usual to make lead wire by extrusion, but it can be drawn if the dies are chosen so that the reduction in cross-sectional area of the

* Communicated by the Author.

wire by the passage through each die is not more than 5 per cent. In order to anneal the wire after passage through a die it is merely left some days at room temperature, when it anneals spontaneously. In making experiments on the elongation of lead wires under tension Andrade⁽²⁾ remarks "Lead anneals itself at room temperature, *i. e.*, lead wires left unstrained for a considerable time always give consistent results." In order to anneal the wire used in these experiments it was stored for three months before the experiments were begun.

Experiments not now discussed showed that the material could be further annealed by boiling in water for some considerable time; this may be exaggerated crystal growth.

Tension.

When a wire of a ductile metal is loaded beyond its elastic limit the nature of its elongation depends on the load; if the elastic limit is not greatly exceeded the rate of elongation is fairly large at first, but rapidly diminishes to zero; this is termed by Andrade "beta flow." For large tensions, however, a viscous flow superimposes itself upon this beta flow, and the wire extends indefinitely until it breaks. With copper, silver, nickel, and to a less extent aluminium and gold, tensions considerably above the elastic limit produce little beta flow and no viscous flow, but in lead the viscous flow predominates throughout and is quite apparent for tensions not greatly in excess of the elastic limit (given by Tammann as 25 kg. per sq. cm.).

Andrade found that severe torsion puts a lead wire in a state to flow viscously from the start. In confirmation of Andrade's observation it was found that in a severely twisted wire some elongation was observable even under a load as small as 10 kg. per sq. cm. Experiments showed, however, that for a tension of 40 kg. per sq. cm. the elongation of a wire initially severely twisted was only 0.14 per cent. after 12 hours. The duration of each experiment is not more than three hours, so that the elongation in an ordinary experiment will certainly be less than this. For a tension of 50 kg. per sq. cm. the elongation in similar circumstances is considerably more than this. In view of the pronounced effect of the rate of twisting upon the elongation of lead discussed in the

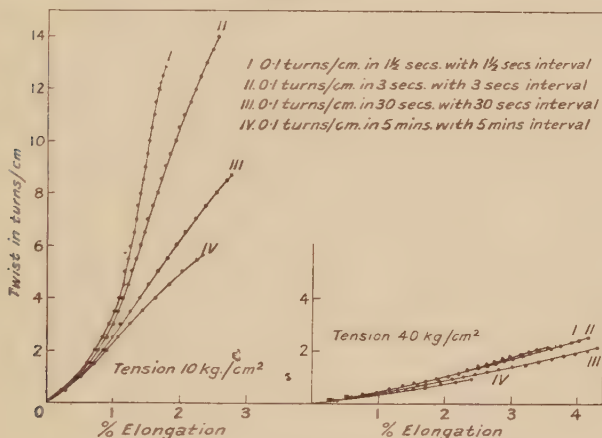
next section, the elongation due to the load within the range 10–40 kg. per sq. cm. is considered to be unimportant. This range was not exceeded in these experiments.

Effect of Rate of Twisting on the Elongation.

The elongation of the wires previously examined (other than those of lead) produced by a given twist and tension is independent of the rate of twisting convenient in turning by hand against a metronome. (The behaviour of these metals using very high or very low rates of twisting has not been studied.)

Fig. 1.

Effect of Rate of Twisting on Elongation of Lead Wire of Diameter 0.075 cm.



Experiments showed, however, that the elongation of lead wires for a given twist is markedly dependent upon the rate of twisting. In these experiments lead wires of diameter 0.075 cm. and length 1 metre under tensions of 10 and 40 kg. per sq. cm. were twisted by increments of 0.1 turn per cm. in 1.5 seconds with 1.5 secs. pause, in 3 secs. with 3 secs. pause, and in 5 minutes with 5 mins. pause between each increment. The results, given in fig. 1, show that the elongation is less the greater the rate of twisting, and that for elongations greater than 0.6 per cent. the dependence of the elongation on the rate of twisting is more pronounced for the small tension than for the large tension.

Norbury's Work on Lead ⁽³⁾.

Most metals are hardened by "cold working," i. e., by deformation of any kind without the application of heat. Twisting under tension is a form of cold working, and the wires of copper, silver, nickel, and gold were rendered sensibly harder and more rigid by twisting; this hardness persists with these metals because they do not begin to anneal below 200–300° C.

Lead is also hardened by cold working, but at room temperature begins to anneal at once. In experiments made by Norbury test blocks of lead were cold worked by hammering, and the Brinell ball hardness was measured at successive intervals of time after the hammering; the amount of cold work was expressed as the percentage reduction of thickness of the block.

The hardness of the lead before hammering was 4.2; it rose to 6 or 7 immediately after the hammering, and then fell again, the fall being the more rapid the greater the amount of cold work; for 10 per cent. cold work the hardness was still 4.6 after 18 hours, whereas for 60 per cent. cold work it had fallen to 4.3 in the first minute; for 50 and 60 per cent. cold work the hardness after 20 seconds was little more than the hardness before hammering. Norbury states that lead when very severely hammered shows no increase in hardness, and finds that it becomes dead soft in 20 seconds after hammering.

Bearing of Norbury's Results on the present Experiments.

This rapid decrease in the hardness of lead produced by cold working, made evident by Norbury's results, serves as a basis for explaining the behaviour of lead wires on twisting.

Experiments have shown that the elongation of imperfectly annealed copper and silver wires for a given twist and tension is less than if the wires are more thoroughly annealed—in fact, for small loads hard drawn copper shows a contraction of as much as 1 per cent. on severe twisting.

In general annealing is known to soften a metal—the more thorough the annealing the softer the metal. It seems reasonable to assume that the larger the elongation

of a lead wire under a given twist the smaller was the mean hardness of the wire during the elongation.

During the operation of twisting a lead wire the increase of hardness due to the twisting is opposed by the self-annealing of the wire, which, as Norbury has shown, can be exceedingly rapid. Thus it is to be expected that the more rapid the twisting, and the shorter the time interval between successive increments of twist, the higher will be the mean hardness of the wire, and the smaller the elongation for a given twist and tension; this is what is actually observed (fig. 1).

Effect of Tension.

Again, in fig. 1 it is seen that the dependence of the elongation upon the rate of twisting is less for a tension of 40 kg. per sq. cm. than for a tension of 10 kg. per sq. cm., the elongation itself being much the greater for the larger tension.

Assuming that this elongation is a form of cold work, then, the rates of twisting being the same, the mean hardness of the wire will be the less for the higher tension, since, as Norbury has shown, the annealing is the more rapid the greater is the amount of cold work.

From this it follows that the mean hardness being the less for the higher tension, and thus tending more closely to the hardness before twisting, differences between the mean hardnesses of the wires for the various rates of twisting will be less for the higher tension than the corresponding differences for the lower tension, and thus, the elongation being a function of the hardness, the elongation will be less dependent on the rate of twisting for the higher tension than for the lower tension. This is what is actually observed (fig. 1).

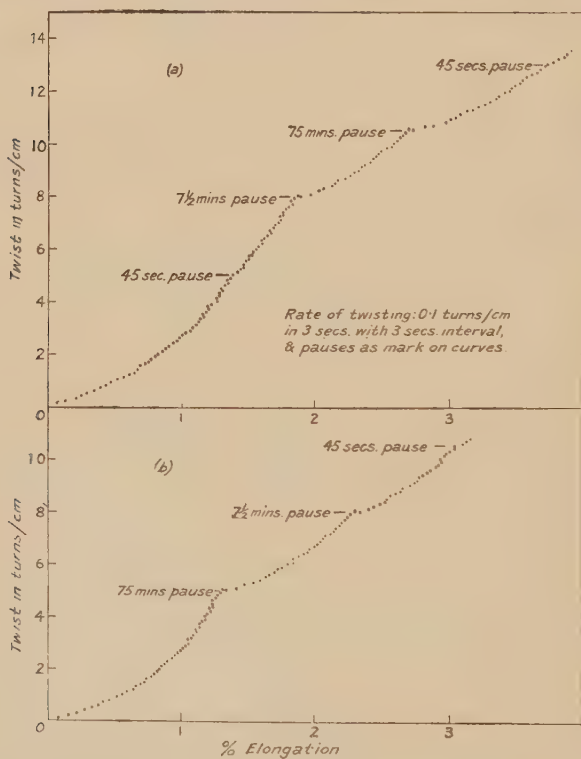
In the same way, if the mean hardness during twisting is greater for the wires under the lower tension, the initial increase in hardness when the first few increments of twist are put in will also be the greater, and the corresponding rate of elongation will decrease more markedly for the lower tension. Thus it is to be expected that the initial parts of the twist-elongation curves for the lower tension will be more markedly concave towards the twist axis than the curves for the higher tension. It will be seen in figs. 1, 2, and 3 that this is the case.

Effect of Pauses during the Twisting.

In the previous experiments just described on lead each of a number of equal increments of twist was put in in 30 seconds, and there was a 30 second pause between each increment. The following experiments show the effect of introducing longer pauses into the twisting.

Fig. 2.

Self-annealing Effect shown by Lead Wire of Diameter 0.075 cm.
under Tension of 10 kg./cm.².



Two lead wires, diameter 0.075 cm., length 1 metre, were twisted under an initial tension of 10 kg. per sq. cm., twisting at the rate of 0.1 turns per cm. in 3 seconds, with 3 seconds pause between each increment. After 5 turns per cm., 8 turns per cm., and 10.5 turns per cm. had been put into the wire a pause was made in the twisting to enable the wire to anneal; in the first wire the successive

pauses were 45 seconds, 7.5 minutes (450 seconds), and 75 minutes; in the second wire they were 75 minutes, 7.5 minutes, and 45 seconds. The results are shown graphically in fig. 2.

The initial portion of both wires shows the concavity towards the twist axis due to the initial hardening. For the first wire the distinct change in curvature after the

Fig. 3.

Twist-extension Curves for chemically pure Lead Wire of various Diameters under various Tensions. Rate of twist 0.1 turn/cm./min.



(The number at the top of each curve is the initial tension on the wire in kg./cm.².)

45 seconds pause shows that some softening has taken place during this pause; this is shown less clearly for the second wire.

In both wires the change in curvature is greatest after the 75 minutes pause, and almost reduplicates the initial concavity due to the initial hardening, indicating that a considerable amount of self annealing has taken place during this pause. The curves indicate that a less but

quite considerable amount of self annealing has taken place after the 7.5 minutes pause.

Effect of Diameter.

It was not considered necessary to study this matter in great detail in lead owing to the complication due to the rapid annealing after work hardening, already discussed; it was considered sufficient to discover whether the general diameter relationships found for copper, silver, and the other metals studied reveal themselves in the case of lead.

Experiments were made on wires of diameter 0.279 cm., 0.157 cm., and 0.075 cm., under tensions of 10, 20, 30, and 40 kg. per sq. cm.; in each experiment the twist was put in in increments of 0.1 turn per cm. in 30 seconds, with 30 seconds pause between each two increments.

Some of the results are shown graphically in fig. 3, where the device of adjusting the scales of twist so as to be in the ratio of the diameters (4:2:1) has been employed; the similarity of the three sets of curves indicates that the twist required to cause a given elongation is inversely proportional to the diameter of the wire.

The results are represented fairly well by the simple formula $e = DT(bt)$, where e is the elongation as a percentage of the initial length, D is the diameter in cm., T the twist in turns per cm. of the initial length, t the tension kg. per sq. cm., and b is a constant.

The fit of this expression to the experimental results is shown in the table.

For each diameter the fit of the expression under the different tensions is fair, except under the lowest tension, where the fit is poor.

The effect of tension is very much greater in the case of lead than in the case of the other metals studied; the mean value of b is about 0.5.

Stress is not laid upon the actual numerical value of b for lead, because this will change with change in the rate of twisting; it is not considered necessary to make experiments or calculations with the object of obtaining an expression which would include the rate of twisting as a variable, since the results show that when the complications due to its self-annealing are allowed for, lead follows the general behaviour of the other metals studied in the face-centred cubic class.

Under the microscope the surface of a twisted lead wire is seen to be distorted into spiral folds; the appearance is rather different from that shown by wires of the other metals studied.

Experimental Values of the Constant b for Lead in the
Expression $e=DT(bt)$.

(Notation as in text above.)

$D=0.293$ cm.

L (gms.) ...	675	...	1350	...	2025	...	2700
t ...	10	...	20	...	30	...	40
DT.	b .		b .		b .		b .
0.15	0.61		0.64		0.59		0.66
0.30	0.47				0.54		
0.45	0.38						
0.60	0.33						
0.78	0.31						
0.90	0.30						
1.05	0.29						
1.20	0.28						

$D=0.197$ cm.

L (gms.) ...	193	...	386	...	579	...	772
t ...	10	...	20	...	30	...	40
DT.	b .		b .		b .		b .
0.15	0.60		0.65		0.72		0.71
0.30	0.49		0.56				
0.45	0.42		0.51				
0.60	0.38						
0.75	0.37						
0.90	0.36						

$D=0.075$ cm.

L (gms.) ...	45	...	89	...	134	...	178
t ...	10	...	20	...	30	...	40
DT.	b .		b .		b .		b .
0.15	0.59		0.59		0.66		0.69
0.30	0.47		0.51				
0.45	0.43						
0.60	0.42						

The experiments which initiated this work were carried out at the suggestion of W. S. Denham, D.Sc., F.I.C., Director of Research of the British Silk Research Association, to whom I am greatly indebted for helpful advice.

Summary.

The paper gives measurements of the elongations which occur when lead wires are twisted under small tensions.

These elongations are considerably greater than those observed in the other common metals of the face-centred cubic class, and for lead unlike these other metals, the elongation for a given twist and tension depends on the *rate* of twisting.

This dependence is linked up with the self-annealing of cold-worked lead at room temperatures.

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- (1) Smythe, "Lead," 'Monographs on Industrial Chemistry,' Longmans Green and Co. (London, 1923).
- (2) Andrade, Proc. Roy. Soc. A, xc. p. 338 (1914).
- (3) Norbury, Trans. Faraday Soc. xix. p. 140 (1923-24).

CI. *The Stability of some Discontinuous Fluid Motions.*
By W. G. L. SUTTON*.

1. CONSIDER the steady two-dimensional motion of a perfect fluid in which there is a line of discontinuity across which there is no transference of fluid and whose equation referred to rectangular cartesian axes is $F_0(x, y) = 0$. Let Q_0 be any point (x_0, y_0) on a portion of the line which is analytic and free from points of inflexion, and let D, D' denote the regions on either side, D' containing the centres of curvature and $F_0(x, y)$ being positive in D . The normal Q_0N drawn *into* D then has direction cosines

$$\frac{1}{K_0} \frac{\partial F_0}{\partial x_0}, \quad \frac{1}{K_0} \frac{\partial F_0}{\partial y_0},$$

where

$$K_0 = + \sqrt{\left\{ \left(\frac{\partial F_0}{\partial x_0} \right)^2 + \left(\frac{\partial F_0}{\partial y_0} \right)^2 \right\}}.$$

Let Q_0T , the tangent at Q_0 drawn so that Q_0T, Q_0N are congruent with Ox, Oy , make an angle θ with Ox where θ is measured in the direction $x \rightarrow y$. We then have

$$\frac{\partial F_0}{\partial x_0} = -K_0 \sin \theta, \quad \frac{\partial F_0}{\partial y_0} = K_0 \cos \theta, \quad . \quad . \quad (1)$$

* Communicated by the Author.

$$\frac{\partial \theta}{\partial s} = -\frac{1}{\rho}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where $\frac{\partial}{\partial s}$ denotes differentiation along the curve and ρ is the numerical radius of curvature.

Now suppose that in consequence of a small disturbance the line of discontinuity becomes

$$F_0(x, y) + F(x, y, t) = 0, \quad . \quad . \quad . \quad . \quad (3)$$

where F is initially small of the first order. Let the old normal at Q_0 meet (3) in Q , whose coordinates referred to Q_0T, Q_0N are $(0, \nu)$. Referred to xOy , Q is then $(x_0 - \nu \sin \theta, y_0 + \nu \cos \theta)$, whence, substituting in (3) and using (1), we get to the first order

$$\nu = -F(x_0, y_0, t)/K_0. \quad . \quad . \quad . \quad . \quad (4)$$

In the old steady motion of D let σ be the density, p_0 the pressure, u_0, v_0 the component x and y velocities, and ψ_0 the stream-function, such that $\frac{\partial \psi_0}{\partial x} = -v_0, \frac{\partial \psi_0}{\partial y} = u_0$, and ψ_0 is constant on $F_0=0$. If the motion is irrotational let ϕ_0 be the velocity-potential, so that $\frac{\partial \phi_0}{\partial x} = u_0, \frac{\partial \phi_0}{\partial y} = v_0$; if rotational we assume that the vorticity $\zeta_0 = -\nabla^2 \psi_0$ is uniform. Under these conditions the fluid velocity along $F_0=0$ in the old motion is constant and wholly tangential. Let it be V_0 in the direction Q_0T . The constant pressure along $F_0=0$ equals P_0 .

In the disturbed motion of D let the component velocities be $u_0 + u, v_0 + v$, the new pressure $p_0 + p$, and the new stream-function $\psi_0 + \psi$. The disturbance is assumed to be irrotational, defined by a velocity potential ϕ . Corresponding quantities for D' are distinguished by dashes.

2. The general equations satisfied by the disturbance arise from the kinematical conditions that $F_0 + F=0$ is a boundary for D and D' and the dynamical condition that the pressure is continuous across the new boundary. Consider first the fluid D . The boundary condition is

$$\frac{\partial F}{\partial t} + (u_0 + u)\left(\frac{\partial F_0}{\partial x} + \frac{\partial F}{\partial x}\right) + (v_0 + v)\left(\frac{\partial F_0}{\partial y} + \frac{\partial F}{\partial y}\right) = 0, \quad (5)$$

to hold along $F_0 + F=0$. Now $u_0 \frac{\partial F_0}{\partial x} + v_0 \frac{\partial F_0}{\partial y}$ will vanish identically if the curves $F_0 = \text{constant}$ coincide with

$\psi_0 = \text{constant}$. We assume this is so, and to the first order (5) then becomes on reduction

$$\frac{\partial F}{\partial t} + V_0 \frac{\partial F}{\partial s} + K_0 \frac{\partial \phi}{\partial n} = 0, \quad . \quad . \quad . \quad (6)$$

to hold along the *old* line $F_0 = 0$, to which ds and dn refer.

Similarly the fluid D' yields the condition

$$\frac{\partial F}{\partial t} + V_0' \frac{\partial F}{\partial s} + K_0 \frac{\partial \phi'}{\partial n} = 0, \quad . \quad . \quad . \quad (6')$$

holding along $F_0 = 0$.

To obtain the pressure condition we have, considering D , pressure at $Q = P_0 + \nu \left(\frac{\partial p_0}{\partial n} \right)_{Q_0} + (p)_{Q_0}$ to the first order.

Now the equations of the *steady* motion are

$$\begin{aligned} v_0 \zeta_0 &= \frac{\partial}{\partial x} \left(\frac{p_0}{\sigma} + \frac{V^2}{2} \right) \\ -u_0 \zeta_0 &= \frac{\partial}{\partial y} \left(\frac{p_0}{\sigma} + \frac{V^2}{2} \right), \end{aligned}$$

where $V^2 = u_0^2 + v_0^2$. Multiplying respectively by $-\sin \theta$, $\cos \theta$, adding and evaluating at Q_0 , we get

$$-V_0 \zeta_0 = \frac{1}{\sigma} \left(\frac{\partial p_0}{\partial n} \right)_{Q_0} + V_0 \left(\frac{\partial V}{\partial n} \right)_{Q_0}.$$

Now

$$\left(\frac{\partial V}{\partial n} \right)_{Q_0} = \left[\frac{\partial}{\partial n} \left(\frac{\partial \psi_0}{\partial n} \right) \right]_{Q_0} = \left[\frac{\partial}{\partial n} \left(\frac{\partial \psi_0}{\partial n} \right) + \frac{\partial}{\partial s} \left(\frac{\partial \psi_0}{\partial s} \right) \right]_{Q_0},$$

since ψ_0 is constant along $F_0 = 0$. Using $\nabla^2 \psi_0 = -\zeta_0$, this gives

$$\begin{aligned} \left(\frac{\partial V}{\partial n} \right)_{Q_0} &= -\zeta_0 + \left[\frac{\partial \theta}{\partial x} \cdot \frac{\partial \psi_0}{\partial y} - \frac{\partial \theta}{\partial y} \cdot \frac{\partial \psi_0}{\partial x} \right]_{Q_0} \\ &= -\zeta_0 - V_0/\rho, \quad \text{from (2).} \end{aligned}$$

Hence

$$\nu \left(\frac{\partial p_0}{\partial n} \right)_{Q_0} = \nu \frac{V_0^2}{\rho} = - \left[\frac{F \cdot V_0^2}{K_0 \rho} \right]_{Q_0}, \quad \text{from (4).}$$

[This result is almost intuitive if we note that in the *old steady* motion the pressure-gradient normal to the path of fluid particles must provide the acceleration V_0^2/ρ per unit mass.]

Again, to the first order the equations of the disturbance yield

$$\begin{aligned} -\frac{\partial}{\partial x}\left\{\frac{p}{\sigma} + uu_0 + vv_0\right\} &= \frac{\partial u}{\partial t} - v\xi_0 = \frac{\partial}{\partial x}\left\{\frac{\partial\phi}{\partial t} + \xi_0\psi\right\} \\ -\frac{\partial}{\partial y}\left\{\frac{p}{\sigma} + uu_0 + vv_0\right\} &= \frac{\partial v}{\partial t} + u\xi_0 = \frac{\partial}{\partial y}\left\{\frac{\partial\phi}{\partial t} + \xi_0\psi\right\} \end{aligned}$$

Integrating and absorbing the additive function of t in the terms on the right we obtain

$$\left(\frac{p}{\sigma}\right)_{Q_0} = \left[-\frac{\partial\phi}{\partial t} - \xi_0\psi - V_0\frac{\partial\phi}{\partial s}\right]_{Q_0}.$$

Hence, finally, the pressure at Q evaluated from D is

$$P_0 - \sigma \left[\frac{\partial\phi}{\partial t} + V_0 \frac{\partial\phi}{\partial s} + \frac{FV_0^2}{K_0\rho} + \xi_0\psi \right]_{Q_0}.$$

Considering similarly the fluid D' , and noting that $P_0 = P'_0$, we obtain the required condition as

$$\begin{aligned} \sigma \left[\frac{\partial\phi}{\partial t} + V_0 \frac{\partial\phi}{\partial s} + \frac{FV_0^2}{K_0\rho} + \xi_0\psi \right] \\ = \sigma' \left[\frac{\partial\phi'}{\partial t} + V'_0 \frac{\partial\phi'}{\partial s} + \frac{FV'^2}{K'_0\rho} + \xi'_0\psi' \right], \quad \dots (7) \end{aligned}$$

to hold along $F_0 = 0$. (6), (6'), (7) are the general equations required.

3. The above equations may be applied to yield the known results about the stability of a circular vortex* and the instability of a single rectilinear jet†. Cisotti‡ has shown that the only other configuration possible for a single (two-dimensional) jet moving through quiescent fluid is when the bounding lines of discontinuity are concentric circles. The velocity at any point of the jet is then purely transversal and inversely proportional to the distance of the point from the centre of the circles.

To discuss the stability of this case let (r, η) be polar coordinates with the steady velocity in the direction of η increasing. There are two lines of discontinuity given by $r = a$, $r = b > a$. Let ϕ_1, ϕ, ϕ_2 be the velocity-potentials of the disturbances produced in the regions $r < a$, $a < r < b$, and $r > b$ respectively. The corresponding densities are $\sigma_1, \sigma, \sigma_2$. In equations (6), (6'), (7) we write $F_0 \equiv r - a$, so

* *E. g.*, Lamb, 'Hydrodynamics,' ch. vii.

† Rayleigh, "On the Instability of Jets," Proc. Lond. Math. Soc. x. (1879) Papers, i. 58, p. 369.

‡ 'Idromeccanica Piana,' i. ch. iii.

that $K_0=1$, $dn=dr$, $ds=-ad\eta$, $V_0'=0$, $V_0=-a\omega_1$, where $\omega_1 > 0$ and $\phi'=\phi_1$. The jet is region D. The disturbance on $r=a$ being defined by $F_1(\eta, t)$, we obtain the equations

$$\left. \begin{aligned} \frac{\partial F_1}{\partial t} + \omega_1 \frac{\partial F_1}{\partial \eta} + \frac{\partial \phi}{\partial r} &= 0 = \frac{\partial F_1}{\partial t} + \frac{\partial \phi_1}{\partial r}, \\ \frac{\partial \phi}{\partial t} + \omega_1 \frac{\partial \phi}{\partial \eta} + F_1 a \omega_1^2 &= \frac{\sigma_1}{\sigma} \frac{\partial \phi_1}{\partial t}, \end{aligned} \right\}, \dots (8)$$

to hold at $r=a$. For the line $r=b$ the jet must be taken as the region D', and if the disturbance on $r=b$ is defined by $F_2(\eta, t)$ the equations obtained are (8) with the suffix 1 replaced by 2 and a by b , where the velocity along $r=b$ is $b\omega_2$. Cisotti's result for the velocities gives $\omega_2=\gamma\omega_1$, where $\gamma=a^2/b^2 < 1$. Now assume for the new boundaries

$$\begin{aligned} r-a + \frac{C_1}{a} \sin(m\eta - nt) &= 0, \\ r-b + \frac{C_2}{b} \sin(m\eta - nt) &= 0. \end{aligned}$$

If m is integral the conditions demanded by re-entrance and incompressibility are satisfied to the first order. Taking $m > 0$, we assume $\phi_1=A_1 r^m \cos(m\eta - nt)$,

$$\phi_2=B_2 r^{-m} \cos(m\eta - nt), \quad \phi=(Ar^m + Br^{-m}) \cos(m\eta - nt).$$

We first obtain $A_1=\omega C_1 a^{-m}$; $B_2=-\omega C_2 b^m$, where $\omega=n/m$. The elimination of the ratios $A:B:C_1:C_2$ from the remaining conditions then gives the equation

$$\begin{aligned} \{(\omega - \omega_1)^2 + \sigma_1 \omega^2 / \sigma - \omega_1^2 / m\} \{(\omega - \omega_2)^2 + \sigma_2 \omega^2 / \sigma + \omega_2^2 / m\} \\ = \gamma^m \{(\omega - \omega_1)^2 - \sigma_1 \omega^2 / \sigma + \omega_1^2 / m\} \{(\omega - \omega_2)^2 - \sigma_2 \omega^2 / \sigma - \omega_2^2 / m\} \end{aligned} \dots (9)$$

From this equation we deduce two general results. In the first place we can show that if $\sigma_1 \neq 0$ there is instability in general, for, in virtue of Fourier's theorem, the general disturbance can be stable only if (9) furnishes a value of ω real for all positive integral values of m . Now since $0 < \gamma < 1$, the right member of (9) can be made arbitrarily small by taking m sufficiently large; further, the second factor on the left is essentially positive. A necessary condition for stability is, therefore, that the first factor on the left of (9) should split up into real linear factors. This gives

$$\left(1 - \frac{1}{m}\right) \left(1 + \frac{\sigma_1}{\sigma}\right) \leq 1,$$

which is certainly not satisfied for sufficiently large m .

If, however, $\sigma_1 = 0$, there are two velocities of propagation of a stable disturbance, for, writing (9) as $f(\omega) = 0$, we have in this case $f(\infty) > 0$, $f(\omega_1) < 0$, and $f(0) > 0$ if $m > 1$, so that for $m > 1$ equation (9) has certainly two real roots, and for m sufficiently large it is clear that there are only two real roots. Further, if $m = 1$ equation (9) has two zero roots, and with $\sigma_1 = 0$ it is easily shown that the remaining roots are also real.

A few particular results may be noted. Putting $\sigma_1 = \sigma = \sigma_2$ and $m = 1$, we get $\omega = \omega_1$ or $\omega = \omega_2$, so that this particular disturbance of a jet moving in fluid of its own density is stable. Again, for the case of a hollow vortex in an infinite fluid moving irrotationally we put $\gamma = 0 = \sigma_1$, and obtain the known result * that the disturbance is stable and travels with

angular velocity $\omega = \omega_1 \left(1 \pm \frac{1}{\sqrt{m}}\right)$.

4. The tractability of the preceding cases arises from the constancy of ρ along the line of discontinuity, so that the coefficients in the boundary conditions are constant. Rayleigh, in the paper mentioned, deals with the case of a curved boundary, with the remark that it can be reduced to the rectilinear case in new variables ϕ_0, ψ_0 . The term containing ρ in (7), however, makes this extension impossible.

Summary.

The general equations to be satisfied by a disturbance along a line of discontinuity in the two-dimensional motion of a perfect fluid are obtained, and applied to discuss the stability of a single jet bounded by concentric circles.

CII. *On the Periodic Classification of the Elements.* By
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Introduction.

THE periodic table of elements, formerly of interest only to chemists, has gained in importance in recent times owing to its close association with the electronic structure of the atoms. In fact the periodicity of the

* Hicks, Phil. Trans. Roy. Soc. 1884; Kelvin, Papers, iv. p. 158.

† Communicated by the Author.

chemical and physical properties of the elements can be traced to a periodicity in the nature of the grouping of the electrons round the atomic core. As has been remarked by Sommerfeld *, there is a "phylogenetic principle" for atoms just as much as in biology. According to this principle there is a periodic reappearance of certain typical configurations (*e. g.*, the helium configuration or the neon configuration) in the outermost shell of the core as we proceed from element to element. This reappearance of certain typical configurations is necessarily accompanied by a reappearance of certain typical chemical properties. The periodic reappearance of chemical and physical properties has, of course, long been known, and is the basis of the periodic chart first prepared by Mendeléef; but its true significance has only been discovered recently with the increase of our knowledge of the electronic structure of the atoms. Simultaneously with the growth of our knowledge of the periodicity in chemical properties periodicities in other physical properties, *e. g.*, atomic volumes, values of ionization potentials, term values in spectral series, etc., have also been discovered. In fact the chemical and physical properties of all the known elements appear to us now in ordered array, whereas only a couple of decades ago they were more or less in chaos.

The chief function of a periodic chart or table is to depict the physical and chemical properties of the elements as a function of their atomic weights, and also to group together elements belonging to different periods but exhibiting similar properties. Various forms of the chart have, of course, been presented from time to time, each of which shows advantages. The periodic charts or tables may be broadly divided into two classes. In one class a chart is usually given which differs more or less from the conventional chart originally proposed by Mendeléef. Another class which, though not usually known as a periodic chart, nevertheless depicts the periodicity, and to which I would give the name "The Periodic Electron Configuration Table," has been developed only in recent times. In these tables the configurations of the electrons in the various shells and subshells round the atomic nucleus are written down as the atomic number increases from one

* 'Three Lectures of Atomic Physics,' by Arnold Sommerfeld (Methuen & Co. Ltd.), p. 7.

element to another. By comparison with the ordinary chart the nature of the electron configuration which is responsible for a particular type of chemical property can be found. Each of the two types, the periodic chart and the electron configuration table, presents advantages peculiar to itself. In one the chemical grouping of elements of similar kind is clearly shown, and in the other the gradual building-up of the complex atoms of higher atomic weights from comparatively simple elementary structure is shown. To gain a complete knowledge of the group properties and the electronic structure of the elements it is necessary to read the chart and the table side by side.

It occurred to the writer of this note that it would be of great help to students of physics as well as of chemistry if a scheme could be devised which would link up the periodic chart of the chemist with the electron configuration table of the physicist. Such a chart should not only classify the elements into Mendeléef periods and divide them into the well-known groups or families, but also at the same time indicate, without having recourse to a second table, the nature of the electron configuration which is responsible for the periodic classification and the group division. The author of this note has prepared such a modified chart, and he hopes that it serves the double purpose mentioned above.

In what follows the existing charts and tables are first briefly described for comparison, and next the author's modified chart is given and its advantages discussed.

Fig. 1 is a modern adaptation of the periodic chart first proposed by Mendeléef and Lothar Meyer about 1870. In this the elements are written down in horizontal rows in order of increasing atomic weight (with only two exceptions, indicated by arrows in the figure). A row is broken off at a point where the succeeding element shows a marked resemblance to the first element of the row. Another row is now begun with this allied element placed below the first element of the previous row. Proceeding in this way it is found that the number of elements contained in each period is not the same. The first period contains only two elements, the second and third periods eight elements each, the fourth and fifth periods eighteen elements each, while in the sixth period one has to pass over

thirty-two elements before the characteristic properties of the first element reappear. The seventh period, containing only six elements, is not complete. The horizontal rows in the chart thus exhibit the periodicity in the chemical properties of the elements as a function of their atomic weights. The serial numbers of the periods are given in the vertical column in the extreme left. The chart performs another function: it not only classifies the elements in different periods, but also divides them into different groups or families containing elements of similar chemical properties. Eight such groups are generally recognized at present, and the main criterion in the division into groups is the chemical valency of the elements. The

Fig. 1.

	I	II	III	IV	V	VI	VII	VIII	O
1	1H								2He
2	3Li	4Be	5B	6C	7N	8O	9F		10Ne
3	11Na	12Mg	13Al	14Si	15P	16S	17Cl		18Ar
4	19K	20Ca	21Sc	22Ti	23V	24Cr	25Mn	26Fe 27Co 28Ni	
	29Cu	30Zn	31Ga	32Ge	33As	34Se	35Br		36Kr
5	37Rb	38Sr	39Y	40Zr	41Nb	42Mo	43Mn	44Ru 45Rh 46Pd	
	47Ag	48Cd	49In	50Sn	51Sb	52Te	53I		54Xe
6	55Cs	56Ba	RARE EARTHS	72Hf	73Ta	74W	75Re	76Os 77Ir 78Pt	
	79Au	80Hg	81Tl	82Pb	83Bi	84Po	85—		86Rn
7	87—	88Ra	89Ac	90Th	91Pa	92U			

Periodic chart (Mendeléef).

significance of this chart is so well known that any further explanation is unnecessary. It is true that from time to time criticisms have been levelled at the arbitrariness and artificiality of some of its details, yet it cannot be denied that since the time of its discovery by Mendeléef the scheme has been of striking and often of spectacular success in classifying the elements, in predicting their properties, and even in anticipating the existence of unknown elements to fill the gaps in the table. Further, it adduces a strong argument in favour of the theory that the periodic reappearance of the same chemical properties must in some way be connected with a periodicity in the structure of the atom.

Fig. 2 shows the second type of periodic table which has come into use in recent times, and to which I propose to give

the name "The Periodic Electron Configuration Table." The table shows clearly how the total number of electrons (equal to the atomic number of the element) is distributed in the various orbits round the positively charged nucleus

Fig. 2.

Group	K	L	M	N	O	P	Q
Quantum Numbers	1	2, 2 ₁	3, 3 ₁ 3 ₂	4, 4 ₁ 4 ₂ 4 ₃ 4 ₄	5, 5 ₁ 5 ₂ 5 ₃ 5 ₄ 5 ₅	6, 6 ₁ 6 ₂ 6 ₃ 6 ₄ 6 ₅ 6 ₆	7, 7 ₁ 7 ₂ 7 ₃ 7 ₄ 7 ₅ 7 ₆ 7 ₇
Element							
1 H	1						
2 He	2						
3 Li	2	1					
4 Be	2	2					
5 B	2	2	1				
6 C	2	2	2				
7 N	2	2	2	1			
8 O	2	2	2	2			
9 F	2	2	2	2	1		
10 Ne	2	2	2	2	2		
11 Na	2	2	6	1			
12 Mg	2	2	6	2			
13 Al	2	2	6	2	1		
14 Si	2	2	6	2	2		
15 P	2	2	6	2	2	1	
16 S	2	2	6	2	2	2	
17 Cl	2	2	6	2	2	2	1
18 Ar	2	2	6	2	2	2	2
19 K	2	2	6	2	6		
20 Ca	2	2	6	2	6		
21 Sc	2	2	6	2	6	1	
22 Ti	2	2	6	2	6	2	
23 V	2	2	6	2	6	2	1
24 Cr	2	2	6	2	6	2	2
25 Mn	2	2	6	2	6	2	2
26 Fe	2	2	6	2	6	2	2
27 Co	2	2	6	2	6	2	2
28 Ni	2	2	6	2	6	2	2
29 Cu	2	2	6	2	6	10	
30 Zn	2	2	6	2	6	10	
31 Ga	2	2	6	2	6	10	2
32 Ge	2	2	6	2	6	10	2
33 As	2	2	6	2	6	10	2
34 Se	2	2	6	2	6	10	2
35 Br	2	2	6	2	6	10	2
36 Kr	2	2	6	2	6	10	2
37 Rb	2	2	6	2	6	10	
38 Sr	2	2	6	2	6	10	
39 Y	2	2	6	2	6	10	2
40 Zr	2	2	6	2	6	10	2
41 Nb	2	2	6	2	6	10	2
42 Mo	2	2	6	2	6	10	2
43 Tc	2	2	6	2	6	10	2
44 Ru	2	2	6	2	6	10	2
45 Rh	2	2	6	2	6	10	2
46 Pd	2	2	6	2	6	10	2
47 Ag	2	2	6	2	6	10	2
48 Cd	2	2	6	2	6	10	2
49 In	2	2	6	2	6	10	2
50 Sn	2	2	6	2	6	10	2
51 Sb	2	2	6	2	6	10	2
52 Te	2	2	6	2	6	10	2
53 I	2	2	6	2	6	10	2
54 Xe	2	2	6	2	6	10	2
55 Cs	2	2	6	2	6	10	
56 Ba	2	2	6	2	6	10	
57 La	2	2	6	2	6	10	2
58 Ce	2	2	6	2	6	10	2
59 Pr	2	2	6	2	6	10	2
60 Nd	2	2	6	2	6	10	2
61 Pm	2	2	6	2	6	10	2
62 Sm	2	2	6	2	6	10	2
63 Eu	2	2	6	2	6	10	2
64 Gd	2	2	6	2	6	10	2
65 Tb	2	2	6	2	6	10	2
66 Dy	2	2	6	2	6	10	2
67 Ho	2	2	6	2	6	10	2
68 Er	2	2	6	2	6	10	2
69 Tm	2	2	6	2	6	10	2
70 Yb	2	2	6	2	6	10	2
71 Lu	2	2	6	2	6	10	2
72 Hf	2	2	6	2	6	10	2
73 Ta	2	2	6	2	6	10	2
74 W	2	2	6	2	6	10	2
75 Re	2	2	6	2	6	10	2
76 Os	2	2	6	2	6	10	2
77 Ir	2	2	6	2	6	10	2
78 Pt	2	2	6	2	6	10	2
79 Au	2	2	6	2	6	10	2
80 Hg	2	2	6	2	6	10	2
81 Tl	2	2	6	2	6	10	2
82 Pb	2	2	6	2	6	10	2
83 Bi	2	2	6	2	6	10	2
84 Po	2	2	6	2	6	10	2
85 At	2	2	6	2	6	10	2
86 Rn	2	2	6	2	6	10	2
87 Fr	2	2	6	2	6	10	
88 Ra	2	2	6	2	6	10	
89 Ac	2	2	6	2	6	10	2
90 Th	2	2	6	2	6	10	2
91 Pa	2	2	6	2	6	10	2
92 U	2	2	6	2	6	10	2
93 Np	2	2	6	2	6	10	2
94 Pu	2	2	6	2	6	10	2
95 Am	2	2	6	2	6	10	2
96 Cm	2	2	6	2	6	10	2
97 Bk	2	2	6	2	6	10	2
98 Cf	2	2	6	2	6	10	2
99 Es	2	2	6	2	6	10	2
100 Fm	2	2	6	2	6	10	2
101 Md	2	2	6	2	6	10	2
102 No	2	2	6	2	6	10	2
103 Lr	2	2	6	2	6	10	2
104 Rf	2	2	6	2	6	10	2
105 Db	2	2	6	2	6	10	2
106 Sg	2	2	6	2	6	10	2
107 Bh	2	2	6	2	6	10	2
108 Hs	2	2	6	2	6	10	2
109 Mt	2	2	6	2	6	10	2
110 Ds	2	2	6	2	6	10	2
111 Rg	2	2	6	2	6	10	2
112 Uub	2	2	6	2	6	10	2
113 Uut	2	2	6	2	6	10	2
114 Uuq	2	2	6	2	6	10	2
115 Uup	2	2	6	2	6	10	2
116 Uuh	2	2	6	2	6	10	2
117 Uus	2	2	6	2	6	10	2
118 Uuo	2	2	6	2	6	10	2

Electron configuration table.

of the atom. The shape and size of the orbits are determined by the usual quantum conditions. Our extension of knowledge in this direction is largely due to the efforts of Rutherford, Bohr, McLennan, Main Smith, Stoner, and others. Very recently the knowledge has gained in

precision by the "Principle of Exclusion" developed by Pauli, which, taking into account the Goudsmit spin of the electron, states that no two electrons in an atom can have the same set of quantum numbers. The table* given in fig. 2 is the one generally accepted at present. It is a modification of the Main Smith-Stoner scheme, in which for the purpose of X-ray classification each subgroup (M_2 , M_3 , etc.) is further subdivided into grouplets. The utility of such subdivision has been questioned by Saha and Ray† and others, and in the present table Stoner's subdivisions of a single subgroup into grouplets is disregarded. This table, though it shows clearly the gradual building of the heavier atoms by addition of electrons in the various orbits, does not indicate sufficiently clearly the chemical grouping of the elements. For instance, if one wants to know the names of the alkali metals one has to pick out the first element from each division in the table. This is comparatively easy; but if one wants to know the elements allied to, say, N, namely P, As, Sb, and Bi, one has actually to search for them in the table to find out the similarity in the electron grouping in the outermost orbits which is responsible for their characteristic chemical properties. The next chart which we are going to discuss obviates this difficulty, but is defective in another direction.

Fig. 3 gives the periodic chart according to Bohr‡. This chart not only indicates in a general way the distribution of electrons in the various shells as successive elements are built up but also attempts to depict the chemical grouping by joining together elements of markedly similar character with connecting lines. The chart is, however, defective in two respects. In the first place, it does not tell us anything about the distribution of electrons in the various sublevels, for which reference one has to consult tables like those given in fig. 2; and, in the second place, the division of the elements into groups by drawing connecting lines is not satisfactory from the point of view of the chemists. The chief merit of the chart is that it represents graphically the various stages of the process of atom building, and illustrates how the different stages correspond to the different periods.

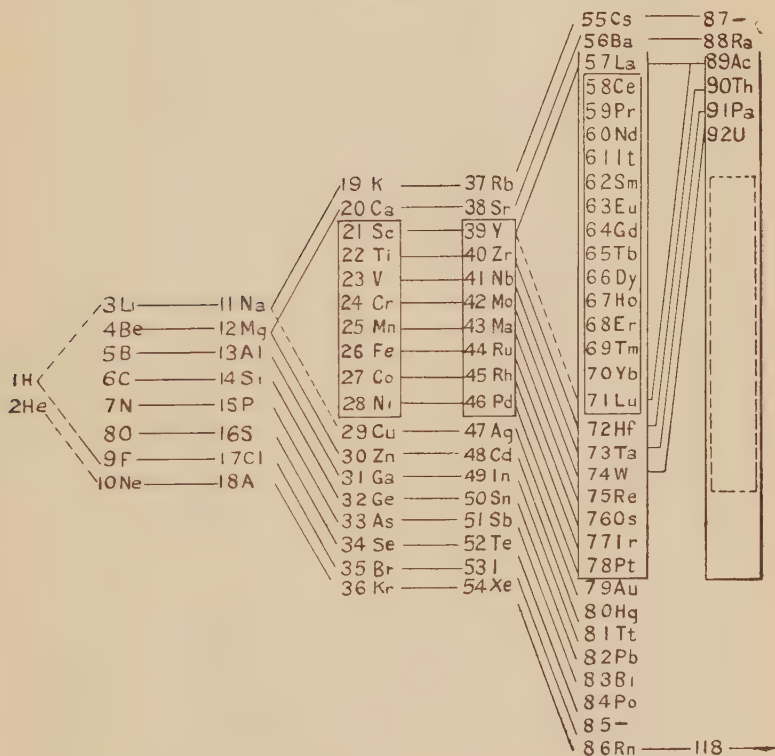
* The table in fig. 2 is taken from Andrade's 'Structure of the Atom,' p. 476, 3rd edition.

Phys. Zeits. xxviii. pp. 221-225 (March 15, 1927).

† 'Nature,' cxii. p. 30 (1923).

The next chart, proposed by Saha * (fig. 4), is remarkable in many ways. The elements are arranged in five regular "groups" (Li to Ne, Na to A, Cu to Kr, Ag to Xe, and Au to Nt or Rn) and in "groups" of the intermediate transitional elements. H and He stand apart. In order to elucidate the optical spectra the various levels, M, N, O, etc., are

Fig. 3.



Periodic chart (Bohr).

written in horizontal rows so staggered that the L_1, N_1, M_1, O_1 , and L_2, M_2, N_2, O_2 , etc. are in parallel diagonals. The use of this chart can best be described in Saha's own words: "The chart explains the formation of (1) the regular groups, (2) the transitional groups, (3) the rare earths, and gives the electronic composition of the outer shells of the

* Phil. Mag. iii. p. 1237 (June 1927).

electrons, as obtained from Hund's interpretation of complex spectra of elements. The distinctive nature of the small groups of two, consisting of an alkali and an alkaline earth, is clearly brought out" (Saha uses the word "group" in the same sense as we have used the word "period"). The main interest in Saha's chart lies in the fact that it enables us to predict the spectral terms belonging to a certain element with comparative ease. In a sense it is also an improvement upon Bohr's chart, inasmuch as it shows the electron grouping in the various sublevels. But it is also defective in the sense that it does not give us the chemical group division of the elements.

Amongst the other charts mention might be made of one due to Rydberg* and of another due to von Antropoff†. In the former the elements are written down in a number of vertical series, all the elements in any one series having the same maximum positive or negative valency. Pairs of series are chosen such that the maximum positive valency of one added to the maximum negative valency of the other gives 8, and these are put in one column. The other table, due to von Antropoff, is merely a modification of the chart given in fig. 1. The elements forming the subgroups, such as Cu, Ag, Au, or Zn, Cd, Hg are placed in different columns, and their relation to the main group is shown in an ingenious manner by drawing lines connecting the main groups with the subgroups. The two charts are not given here, as they would unnecessarily increase the bulk of the paper.

The periodic chart proposed by the writer is given in fig. 5. The chart not only illustrates the periodic classification and the group division of the elements, but also gives the distribution of the electrons in the various shells and subshells round the atomic nucleus. Unlike the conventional periodic chart (fig. 1) elements of the same period are placed in vertical columns instead of in horizontal rows, and elements belonging to the same group or family are placed in horizontal rows instead of in vertical columns. The first horizontal line on the top of the table gives the names of the various levels, or "shells" as they

* For a description of this chart see Andrade, 'The Structure of the Atom,' pp. 472-473, 3rd edition (1927).

† See 'Structure of Line Spectra' by Pauling and Goudsmit (McGraw-Hill Book Co.), p. 146.

are called by X-ray spectroscopists. The second line gives the quantum values of the orbits. An orbit is designated by n_K , where n is the principal quantum number and K the azimuthal quantum number*. The numbers within brackets immediately beneath these indicate the maximum number of electrons which the sublevel under question can hold according to Pauli's principle of exclusion. It follows from this principle that any X level with a principal

Fig. 4.

K ₁		Transition Group I			Transition Group II	Rare Earths	Transition Group III	
1H	2	21Sc	M ₂ 2N ₁ , 2M ₂ N ₁	33V	2N ₁ O ₁ N ₂ 2O ₁	57La	71Lu	89Ac
2He		22Ti	2M ₂ 2N ₁ , 3M ₂ N ₁	40Zr	3N ₁ O ₁ 2N ₂ 2O ₁	58Ce	72Hf	90Th
3Li	L ₁	23V	3M ₂ 2N ₁ , 4M ₂ N ₁	41Nb	4N ₁ O ₁ 3N ₂ 2O ₁	59Pr	73Ta	91Pa
4Be	2L ₁	24Cr	5M ₂ N ₁ , 4M ₂ 2N ₁	42Mo	5N ₁ O ₁ , 4N ₂ 2O ₁	60Nd	74W	92U
5B	2L ₂	25Mn	5M ₂ 2N ₁ , 6M ₂ N ₁	43Mn		61	75Re	
6C	2L ₂	26Fe	6M ₂ 2N ₁ , 7M ₂ N ₁	44Ru	6N ₂ 2O ₁ , 7N ₂ O ₁	62Sm	76Os	
7N	3L ₂	27Co	7M ₂ 2N ₁ , 8M ₂ N ₁	45Rh	8N ₁ O ₁ , 7N ₂ 2O ₁	63Eu	77Ir	
8O	4L ₂	28Ni	8M ₂ 2N ₁ , 9M ₂ N ₁	46Pd	10N ₂ N ₃ O ₁	64Gd	78Pt	
9F	5L ₂					65Tb		
10Ne	6L ₂					66Dy		
11Na	M ₁					67Ho		
12Mg	2M ₁					68Er		
13Al	M ₁					69Tm		
14Si	2M ₂					70Yb		
15P	3M ₂							
16S	4M ₂							
17Cl	5M ₂							
18Ar	6M ₂							
19K	6M ₂ N ₁							
20Ca	6M ₂ 2N ₁							
21Sc								
22Ti								
23V								
24Cr								
25Mn								
26Fe								
27Co								
28Ni								
29Cu								
30Zn								
31Ga								
32Ge								
33As								
34Se								
35Br								
36Kr								
37Rb								
38Sr								
39Y								
40Zr								
41Nb								
42Mo								
43Tc								
44Ru								
45Rh								
46Pd								
47Ag								
48Cd								
49In								
50Sn								
51Sb								
52Te								
53I								
54Xe								
55Cs								
56Ba								
57La								
58Ce								
59Pr								
60Nd								
61Pm								
62Sm								
63Eu								
64Gd								
65Tb								
66Dy								
67Ho								
68Er								
69Tm								
70Yb								
71Lu								
72Hf								
73Ta								
74W								
75Re								
76Os								
77Ir								
78Pt								
79Au								
80Hg								
81Tl								
82Pb								
83Bi								
84Po								
85At								
86Rn								
87Fr								
88Ra								
89Ac								
90Th								
91Pa								
92U								
93Np								
94Pu								
95Am								
96Cm								
97Bk								
98Cf								
99Es								
100Fm								
101Md								
102No								
103Lr								

Periodic chart (Saha).

quantum number n cannot contain more than $2n^2$ electrons, and any X_K sublevel with azimuthal quantum number K cannot contain more than $4l+2$ electrons ($l=K-1$). It will be seen that the number of electrons in the various completed shells or subshells conforms to this rule.

* Alternatively, to emphasize the optical nature of the orbits, we may designate them as $1s$, $2p$, $3d$, etc., 1, 2, 3 standing for the principal quantum numbers and s , p , d corresponding to the azimuthal quantum numbers. For s orbit $K=1$, for p orbit $K=2$, for d orbit $K=3$, and so forth.

The order in which the various sublevels and levels are filled up is of course indicated by the sequence of the atomic numbers (which does not necessarily follow the order of increasing atomic weight), but to emphasize this lines are drawn from element to element, indicating the sequence of the filling process.

We start from 1 H and go down to 2 He filling the K level. Thence we rise to the sublevel L_1 forming the first alkali metal Li. From 3 Li to the inert gas 18 A there is no irregularity in the process of filling. L_1 , L_2 , and M_1 , M_2 sublevels are filled up in proper sequence. The first irregularity is noticed with element 19 K. The joining line from 18 A, instead of going to the space allotted for the first element in the next sublevel M_3 , passes right through this subshell and reaches the N level. The reason of this anomalous behaviour is that the energy of the state with one electron in the M_3 orbit is greater than that of the state with one electron in the N_1 orbit. This is because the L_1 orbits, on account of their high eccentricity, penetrate so deeply into the (Argon) core of the atom that the energy change due to the increase of the principal quantum number from 3 to 4 is overcome. (The greater the difference between the principal and the azimuthal quantum number the greater is the eccentricity of the orbit. The orbit is a circle when the principal quantum number equals the azimuthal quantum number.) This preferential behaviour of the electrons in choosing a level with a higher principal quantum number rather than one with a higher azimuthal quantum number is regularly exhibited in subsequent levels and sublevels. O_1 is preferred to N_3 , P_1 to O_3 , and Q_1 to P_3 . This regularity in the so-called irregularity of the filling process is illustrated in the chart by the joining line from the inert gases 18 A, 36 Kr, 54 Xe, and 86 Rn slanting upwards towards the right and reaching the N, O, P, and Q levels respectively, passing through the M_3 , N_3 , N_4 , O_3 , O_4 , O_5 , and P_3 , P_4 , P_5 , P_6 sublevels.

Having filled the first sublevel of the N shell forming the elements 19 K and 20 Ca the electrons drop back to the M shell and start filling the M_3 sublevel which had been previously left over. This irregularity is repeated between the sublevels O_1 and N_3 , P_1 and O_3 , Q_1 and P_3 , and gives rise to the elements of the so-called transitional periods 21 Sc to 28 Ni, 39 Y to 46 Pd, 57 La to 78 Pt (excepting 58 Ce

Fig. 5.

Group ↓	K 1 (2)	L 2, 2 (2) (6)	M 3, 3 (2) (6)	N 4, 4 (2) (6)	O 5, 5 (2) (6)	P 6, 6 (2) (6)	Q 7, 7 (2) (6)
I	1H	3Li	11Na	19K	37Rb	55Cs	87EkaCs
II	4He	4Be	12Mg	20Ca	38Sr	56Ba	88Ra
III		5B	13Al	21Sc	48Cd	80Hg	89Ac
IV		6C	14Si	22Ti	50Sn	82Pb	90Th
V		7N	15P	23V	51Sb	83Bi	91Pa
VI		8O	16S	24Cr	52Te	84Po	92U
VII		9F	17Cl	25Mn	53I	85EkaI	
VIII	2He	10Ne	18Ar	36Kr	54Xe	86Rn	
Period →	1	2	3	4	5	6	7

Periodic chart (Mitra).

to 71 Lu), and 89 Ac to 92 U. This is clearly depicted in the chart by the left-hand movement of the joining line on the completion of the N_1 , O_1 , and P_1 sublevels, the normal course of the line being up or down and towards the right.

Another type of anomaly when the sublevels M_3 , N_3 , and O_3 near completion is responsible for the formation of a second pair of elements under the sublevels N_1 , O_1 , and P_1 . It arises thus: taking, for example, element 28 in sublevel M_3 , it has its N_1 sublevel complete with 2 electrons and M_3 sublevel incomplete with only 8 electrons, the full quota being 10. It might therefore seem that the next element 29 would have 9 electrons in the M_3 sublevel and 2 electrons in the N_1 sublevel. But instead of that the 29th element completes the M_3 sublevel with 10 electrons, by robbing as it were the N_1 sublevel of one out of its two electrons. Element 29 is thus left with only one valence electron like the alkali metals. Sublevel N_1 is now refilled with another electron, and we get element 30. We thus obtain a fresh pair of elements, 29 and 30, in addition to 19 and 20, with one and two electrons respectively in the N_1 orbits. This happens because the configuration with 9 electrons in the 3_3 orbits and 2 electrons in 4_1 orbits has an energy level a little higher than the normal state, and gives rise to the meta-stable state of the atom. This mode of skipping the last but one orbit of the M_3 sublevel is repeated in N_3 and O_3 sublevels and is responsible for the formation of the elements in the subgroups B of groups I and II (Cu, Ag, Au, and Zn, Cd, Hg). The chart very clearly shows how this anomaly in the filling process towards the end of M_3 , N_3 , and O_3 sublevels gives us eight instead of ten elements in these sublevels and four instead of two elements in the N_1 , O_1 , and P_1 sublevels.

The most interesting part of the scheme is, perhaps, the striking manner in which it illustrates the formation of the rare earths. Up to element 57 the levels and sublevels are filled in the order described just now; N_1 precedes M_3 , O_1 precedes N_3 , and P_1 precedes O_3 . But after 57 La (two electrons in P_1 and one in O_3) the next electron, instead of going to O_3 orbit, which was being filled, prefers to drop back to the N shell and start filling the N_4 sublevel which had been left over during the filling of the N shell. We thus get the array of 14 elements, 58 Ce to 71 Lu, known as rare earths. This is illustrated in the chart by the joining line turning back to the left from element 57 and

reaching the N_4 sublevel by passing through the sublevels O_1 , O_2 . The line returns to its normal position, element 72 Hf, after the sublevel N_4 has received its full complement of 14 electrons. The peculiar position of the rare earths, namely that they are all packed in group VI, indicating that they all possess similar chemical properties, is explained by the fact that in filling up this sublevel the configurations of the outer electron shells (2 in O_1 , 6 in O_2 , 1 in O_3 , and 2 in P_1) remain completely unchanged. It is interesting to note that before the advent of the theory of the electronic structure of the atoms the number of elements forming the rare earth group was uncertain, chemical evidence in this direction being not only meagre but often contradictory, giving rise to a good deal of controversy. In Sommerfeld's 'Atombau' (1st German edition, 1919, p. 55) we find the number of rare earths to be 16, 57 La to 72 Hf. In Andrade's 'Structure of the Atom' (3rd edition, 1927, p. 426) the number of rare earths is given as 15, beginning from 57 La and ending with 71 Lu. In all recent works*, however, only 14 elements, 58 Ce to 71 Lu, are recognized as belonging to the rare earth group. This number has been adopted in the scheme under discussion.

Finally, a few words will not be out of place here to explain the advantages of the mode of periodic classification and group division adopted in this chart.

Each horizontal row contains elements of similar electronic configuration and similar chemical properties. Several such rows showing sufficient resemblance to justify their inclusion under one head are separated out from the rest by two horizontal lines, and constitute a group or family. The chart illustrates how a similarity in chemical properties is brought about by a similarity in electronic configuration of the outer shells and subshells. It is seen that elements exhibiting marked resemblance to one another and put in one row have not only the same number of outer valence electrons, but also the same electron configuration in the outermost level of the core. Thus the elements in the alkali group ($Z=3, 11, 19, 37, 55$), or the alkaline earth group ($Z=4, 12, 20, 38, 56, 88$), or the halogen group ($Z=9, 17, 35, 53$), have not only the same

* For instance, in the excellent periodic chart compiled by Henry D. Hubbard, of the U.S. Bureau of Standards, and published by the W. M. Welch Scientific Company (1930).

number of outer valence electrons but also the same electron configuration in the outermost level of the core. The alkalis, for example, have one valence electron and two outermost sublevels of their cores complete (2 in X_1 and 6 in X_2 sublevels). The elements Cu, Ag, Au, on the other hand, though they have, like the alkalis, one valence electron in an X_1 sublevel, yet have properties differing from those of the alkalis on account of three (instead of two) of the outermost sublevels of the core being complete (2 in X_1 , 6 in X_2 , and 10 in X_3). Thus, though they belong to the same group as the alkalis, yet they are not placed in the same row, but rather in another row below displaced to the right. Similar considerations apply to other rows forming subgroups of the main groups. The procedure adopted in the chart is that elements having similar electron structure both for the valence electrons and for the outermost shell of the core are put in one row, and rows having similarity in the disposition of the valence electrons are collected together in one group.

The periods are shown in vertical columns (instead of in horizontal rows, as is usually done), each column being separated from the others by two dotted vertical lines drawn on either side. It may be noticed that elements under any X_1 and X_2 level form together what is called the regular part of the period, while elements under any X_3 level form the transitional part. The second and third periods are wholly regular, because they have no X_3 level.

The mode of representation adopted in the present scheme thus shows in a very natural way how the periodic classification and group division of chemistry conform to the theory of electronic structure of atoms of modern physics. The chart even illustrates such details as the division of main groups into subgroups and the formation of the irregular transitional periods out of the so-called irregularity in the process of filling the various shells and subshells. It is therefore hoped that the chart will be of use not merely to students of physics but also to students of chemistry.

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Oct. 3, 1930.

CIII. *The Circulation due to a Cylinder rotating in a Viscous Fluid.* By E. G. RICHARDSON, B.A., Ph.D., D.Sc. (University College, London) *.

Introduction.

DURING the course of some experiments † on the Magnus effect, *i. e.*, the cross wind force produced by a cylinder rotating in a *stream* of fluid, it was noted that the lift actually obtained was less than that predicted from the classical theory which presupposes a falling off of circulation from the surface of the cylinder, in the sense $u r = \text{constant}$ (r = radial distance from the axis of the cylinder, u = tangential velocity at the distance in question). A few measurements of the velocity distribution round a cylinder rotating in otherwise stagnant fluid showed that in fact the velocity fell off more rapidly than the above law demanded, especially in the neighbourhood of the cylinder, the condition $u \propto r^{-1}$ being approximately fulfilled at several diameters distance from the surface. On the completion of the work on the Magnus effect the question of the circulation round a cylinder rotating in a viscous fluid seemed worthy of further investigation, the results of which are here presented.

Theoretical Part.

In two-dimensional laminar motion about the axis of the cylinder, if we consider unit depth of fluid parallel to the axis, the rate of shear at a radius r , where the angular velocity is $\omega = r \frac{d\omega}{dr}$, and therefore the force due to friction is $\eta r \frac{d}{dr} \left(\frac{u}{r} \right)$ per unit area (η = viscosity coefficient). Hence the work done in a circumferential displacement l :

$$= \eta r l^2 \frac{d}{dr} \left(\frac{u}{r} \right) = \eta l^2 \left(\frac{du}{dr} + \frac{u}{r} \right). \quad \dots \dots (1)$$

The quantity in brackets is the difference of velocity between two annuli at unit distance apart.

Rayleigh ‡ has pointed out that in rotational motion of this type there will be a difference of centrifugal force at different annuli, which has to be reckoned with in examining

* Communicated by the Author.

† Aero. Research Comm., R. & M. No. T 3049 (1931).

‡ Proc. Roy. Soc. A, xciii. p. 145 (1916), or Papers, vi. p. 447.

the stability of the system. This difference for two annuli at unit distance apart will be

$$\frac{\rho \left[ur \left(\frac{du}{dr} + \frac{u}{r} \right) \right]^2}{r+1} - \frac{\rho u^2}{r} = 2\rho \frac{u}{r} \left(\frac{du}{dr} + \frac{u}{r} \right)$$

approximately per unit volume. In a displacement l the mean difference of centrifugal force will be $\frac{l}{2}$ times this (in virtue of the unit breadth and depth of the volume in question), and therefore the work done against centrifugal force will be

$$\rho l^2 \frac{u}{r} \left(\frac{du}{dr} + \frac{u}{r} \right). \quad . \quad . \quad . \quad . \quad . \quad (2)$$

(1) and (2) together give the energy in the fluid annulus. We may use the principle of minimum energy, enunciated by Kelvin and James Thomson, to find the velocity distribution round the cylinder. Assuming $u = U(r/r_0)^n$, where U is the velocity at the surface of the cylinder of radius r_0 , we then have

$$l^2 \left(\frac{du}{dr} + \frac{u}{r} \right) \left(\eta + \rho \frac{u}{r} \right),$$

a minimum, if

$$(n+1)r^{n-1}(\eta + \rho r^{n-1})$$

is a minimum.

This is in fact 0, if $n = -1$. The "potential" distribution of velocity, $u = U r_0/r$, therefore satisfies this case.

Centrifugal force may, however, cause mixing of the fluid in neighbouring annuli, especially in the boundary layer of the cylinder, where u^2/r is high. If this happens the equation for the frictional energy will no longer be (1), but will be that derived from frictional resistance in turbulent motion.

Osborne Reynolds evaluated this resistance as $\rho \delta u \delta v$, where δu and δv are the deviations from the steady velocities in rectangular directions, induced by the turbulent motion. Further, the disturbance of velocity δv along a radius will be proportional to the tangential deviation δu producing it. The work done for a displacement l is accordingly proportional to

$$\rho l^2 \left(\frac{du}{dr} + \frac{u}{r} \right) \left(\frac{du}{dr} + \frac{u}{r} \right). \quad . \quad . \quad . \quad . \quad . \quad (3)$$

For the factor of proportionality we may select some quantity which will be a function of Reynolds' number (R).

Taking now (2) with (3), we have to choose the velocity distribution so that

$$\rho l^2 \left(\frac{du}{dr} + \frac{u}{r} \right) \left\{ \frac{u}{r} + \frac{1}{f(R)} \left(\frac{du}{dr} + \frac{u}{r} \right) \right\}$$

is a minimum.

Substituting $u = U(r/r_0)^n$, as before, we obtain

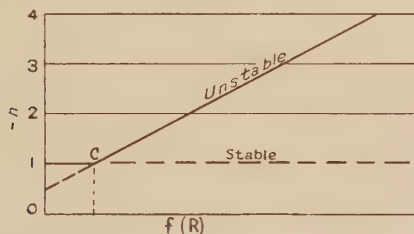
$$r^{2n-2} \rho l^2 \cdot U/r_0^n \{ (n+1) [(f(R) + n+1)] \}.$$

This is 0 if $n = -1$ or $-f(R) - 1$.

In this case it seems natural to take Ur_0/ν for the appropriate Reynolds' number (ν = kinematic viscosity), and write, to a first approximation,

$$f(R) = [Ur_0/\nu]^m.$$

Fig. 1.



The value of R at which this second solution coincides with the first (C on fig. 1) will be the critical value for this type of flow. More exactly there will be a critical velocity for each annulus, and the R in question will be the appropriate value of ur/ν for the given annulus of fluid, rotating with a velocity u at a distance r from the axis.

Experimental Part.

Two cylinders were used, 1 in. and $\frac{1}{2}$ in. diameter respectively, and 2 ft. long, set in journal bearings with the axis vertical in a cubical wooden box (forming part of a wind channel) of 2 ft. side. The hot wire (nickel, 0.002 in. diameter and 2 in. long) was held vertically on a fork mounted on a lathe traverse at the side of the box. Measurements of velocity were made by traversing the hot-wire anemometer outwards in a plane passing through the axis of the cylinder. When the wire was placed near the cylinder a correction had to be applied for the loss of heat by conduction across the intervening film of air. The method of

allowing for this correction has been described elsewhere*, but it is to be noted that the magnitude of the correction was small in these experiments because of the high relative velocity of air and wire close to the cylinder.

Fig. 2.*

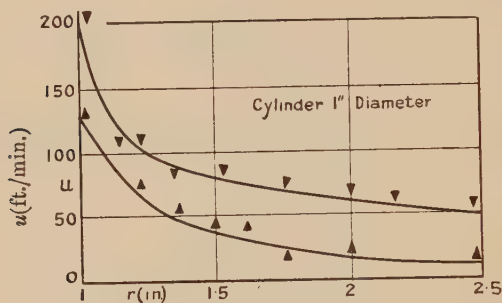
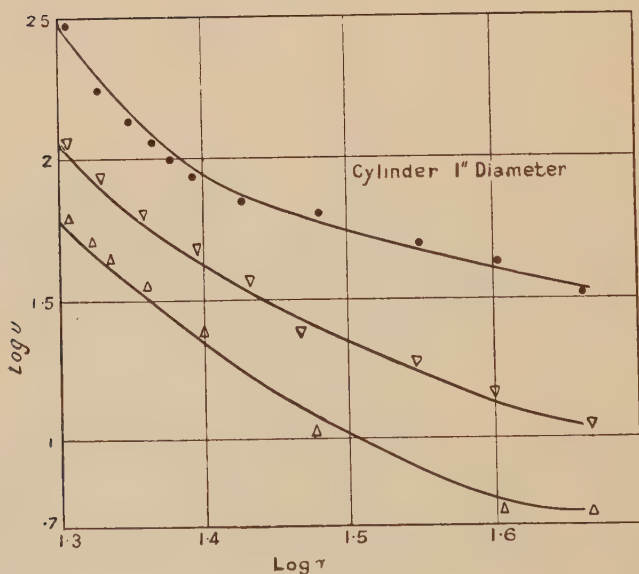


Fig. 3.



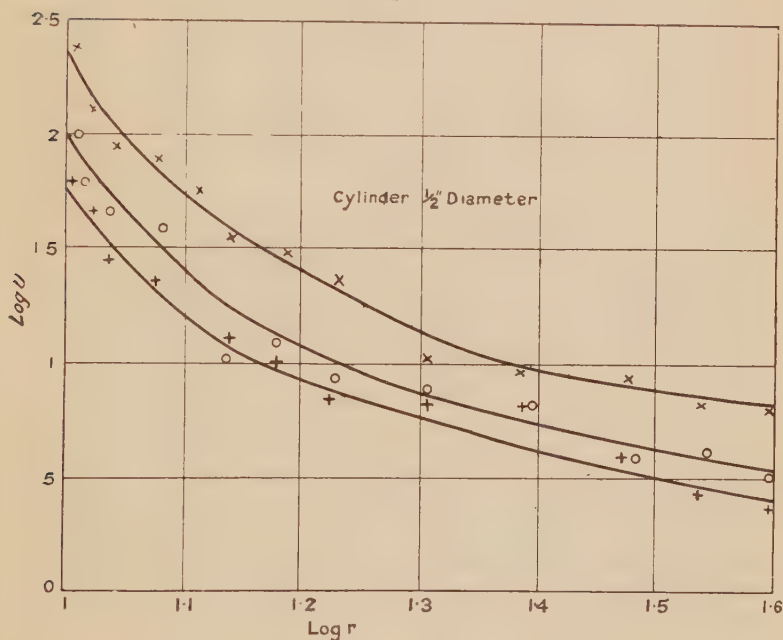
Two typical traverses are shown on fig. 2. The velocity drops rapidly at first, then more slowly. There is thus no fixed value of n for a given cylinder rotated at a given

* Phys. Soc. Proc. xlii. p. 1 (1929).

velocity; n is, in fact, as indicated above, a function of the Reynolds' number of the annulus in question, not of the cylinder itself.

The results for the cylinders are collected in figs. 3 and 4 in the form of a graph of $\log u$ against $\log r$. The slope of these curves at any value of r gives the appropriate value of n . Finally, these values of n are plotted in log form against ur/ν , which is the Reynolds' number of the corresponding annulus, on fig. 5, for comparison with fig. 1.

Fig. 4.

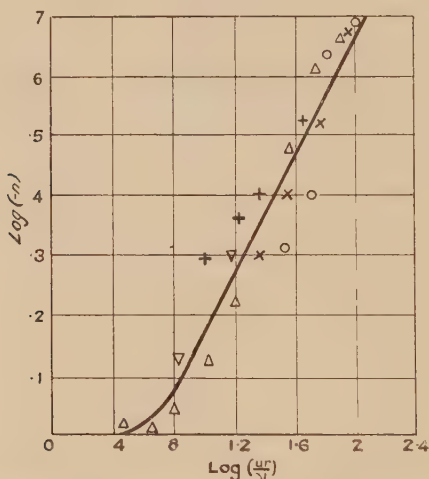


The critical value of ur/ν is about 35, which is about the same as that found in flow past a stationary cylinder for the formation of *Æolian Tones**. This value for the inception of unsteadiness was confirmed by coupling the current through the hot wire by a transformer to a string galvanometer. Actually the average value at which unsteadiness of the string galvanometer appeared was somewhat higher in the case of the rotating cylinder, at about 50 in fact,

* Phys. Soc. Proc. xxxv. p. 153 (1924).

corresponding to a mean value for n of -1.1 , according to fig. 5.

Fig. 5.



In conclusion, I wish to express my best thanks to Prof. E. N. da C. Andrade for his interest and for placing every facility at my disposal during the course of this work.

CIV. *The Magneto-Optical Dispersion of Organic Liquids.*—Part III. *The Magneto-Optical Dispersion of Active Isoamyl Alcohol and Normal Propyl Formate.* By ELFED THOMAS, B.Sc., and Prof. E. J. EVANS, D.Sc., *Physics Department, University College of Swansea**.

INTRODUCTION.

THIS investigation, which is a continuation of previous work carried out by Stephens and Evans† and by Jenkins and Evans‡, is mainly concerned with measurements of the values of Verdet's constants and refractive indices of isoamyl alcohol and propyl formate for wave-lengths in the region of the spectrum extending from about $47\ \mu$ to $32\ \mu$.

The experimental methods employed in these investigations

* Communicated by the Authors.

† Phil. Mag. x, p. 759 (November 1930) [Pt. I.].

‡ Phil. Mag. xi, p. 377 (February 1931) [Pt. II.].

have been described in detail in Part I.*, and consequently only a brief reference will be made to them here. In the magneto-optical measurements the magnetic field was produced by a long solenoid through which a constant current of two amperes was passed, and the quartz tube, 30.5 cm. long, which contained the liquid was placed symmetrically inside it. The value of ΣHL for the columns of liquid employed in these investigations was 12,270 cm. gauss.

The rotations of the plane of polarization for various wave-lengths in the spectrum of the tungsten arc were measured by means of a Bellingham-Stanley polarimeter designed for work in the ultra-violet. The rotations were measured for both directions of the magnetic field, and corrections were made for the rotations due to the quartz ends. Both liquids were tested for natural optical activity, and although the propyl formate was found to be inactive, the isoamyl alcohol showed a fairly large natural rotation due to admixture with lævo-rotary active amyl alcohol. The magnetic rotation of the active isoamyl alcohol for a known wave-length could readily be determined from measurements of the total rotation at that wave-length for both directions of the magnetic field. The results obtained in this way were also confirmed at a few wave-lengths by measuring the natural rotations at these wave-lengths, and also the total rotations when the magnetic field produced a rotation in the same direction as the active substance.

The refractive indices of the two liquids for wave-lengths in the visible were measured by the spectrometer in the usual manner, and for wave-lengths in the ultra-violet by photographing in juxtaposition the spectra of the tungsten arc produced by a hollow prism closed by fused quartz plates, and containing (i.) normal propyl alcohol and (ii.) the liquid under investigation. As the refractive indices of propyl alcohol in the ultra-violet have been measured by Victor Henry†, it is possible, by comparing the two spectra, which slightly overlap one another, to determine the refractive indices of the liquid for various wave-lengths in the ultra-violet.

According to Larmor‡ the value of Verdet's constant is given by the equation

$$\delta = \frac{e}{2mC^2} \lambda \frac{dn}{d\lambda}, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

* *Loc. cit.* [Pt. I.].

† 'Etudes de Photochimie,' p. 61.

‡ 'Æther and Matter,' Appendix F, p. 352.

where C is the velocity of light, $\frac{dn}{d\lambda}$ the dispersion of the substance, and e/m the ratio of charge to mass for all resonators. In deducing this equation the charge e is measured in electrostatic units and the magnetic field in electromagnetic units. Assuming that the natural dispersion of the substance is represented by the following equation of the Ketteler-Helmholtz type :—

$$n^2 - 1 = b_0 + \frac{b_1}{\lambda^2 - \lambda_1^2} + \dots, \quad (2)$$

it can be shown that

$$\phi = n\delta\lambda^2 = K_1 \left[\frac{\lambda^2}{\lambda^2 - \lambda_1^2} \right]^2 + K_2 \left[\frac{\lambda^2}{\lambda^2 - \lambda_2^2} \right]^2 + \dots, \quad (3)^*$$

where K_1 and K_2 are constants and λ_1, λ_2 , etc. for substances transparent in the visible and near infra-red are the wavelengths corresponding to ultra-violet free periods.

For the liquids employed in the present investigation the experimental results can be represented by the equation

$$\phi = n\delta\lambda^2 = K_1 \left[\frac{\lambda^2}{\lambda^2 - \lambda_1^2} \right]^2, \quad (4)$$

involving only one free period in the ultra-violet.

By combining equations (1), (2), and (4) it readily follows that

$$\frac{e}{m} = -\frac{2K_1 C^2}{b_1}, \quad (5)$$

The values of e/m for the resonators in the two liquids can therefore be determined from measurements of the magneto-optical and natural dispersions of the liquids.

EXPERIMENTAL RESULTS.

Natural Dispersion.

The isoamyl alcohol and normal propyl formate employed in this work were the purest obtainable from Dr. Schuchardt of Görlitz, and were subjected to a process of fractional distillation before use. The isoamyl alcohol distilled over at a constant temperature of 131.4°C . at a pressure of 760 mm. of mercury, but in the case of the normal propyl formate only the fraction that distilled over at a temperature

* Richardson, Phil. Mag. xxxi. pp. 232 & 454.

between 80.8 and 81.1° C. at a pressure of 759.8 mm. of mercury was retained for experimental purposes.

Fig. 1.

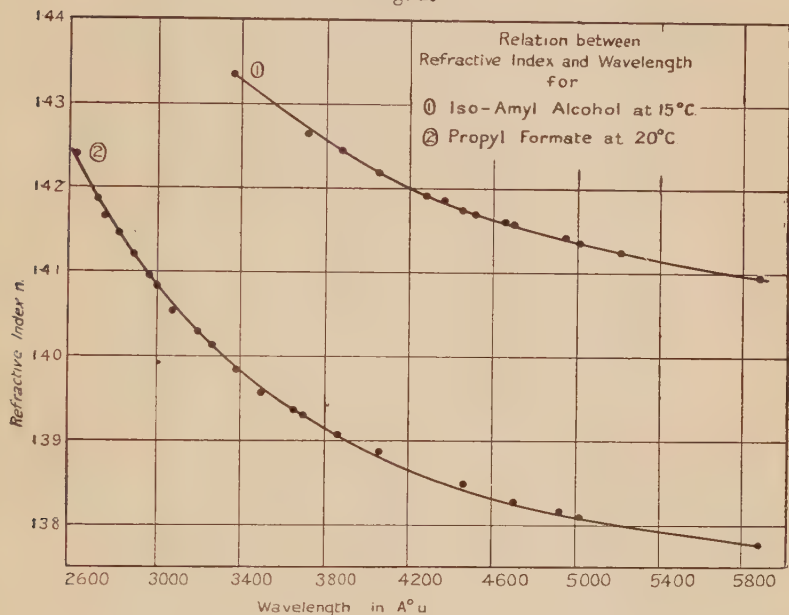


TABLE I. (a).
Isoamyl Alcohol.
Temperature = 15°C.

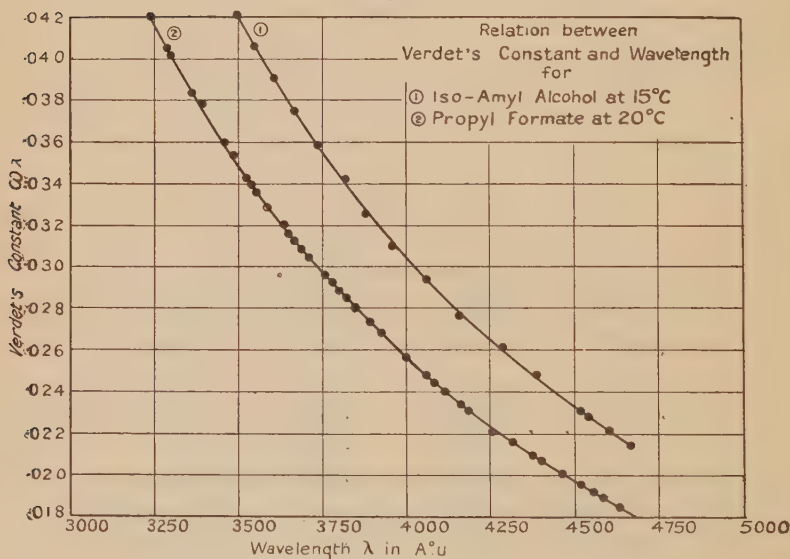
Wave-length (microns).	Refractive index.	Wave-length (microns).	Refractive index.
•6678	1.4072	•4472	1.4175
•5876	1.4096	•4375	1.4183
•5218	1.4123	•4275	1.4192
•5016	1.4135	•4063	1.4217
•4922	1.4141	•3879	1.4242
•4713	1.4157	•3722	1.4264
•4668	1.4160	•3367	1.4334
•4526	1.4171		

The refractive indices of isoamyl alcohol and propyl formate in the visible and ultra-violet regions of the spectrum are given in Tables I. (a) and I. (b) respectively, and plotted in fig. 1.

TABLE I. (b).
Normal Propyl Formate.
Temperature = 20° C.

Wave-length (microns).	Refractive index.	Wave-length (microns).	Refractive index.
·6678	1·3747	·3389	1·3934
·5876	1·3771	·3261	1·4013
·5016	1·3808	·3180	1·4033
·4922	1·3814	·3080	1·4059
·4713	1·3827	·3007	1·4080
·4472	1·3845	·2961	1·4094
·4063	1·3882	·2883	1·4124
·3860	1·3907	·2824	1·4146
·3688	1·3930	·2766	1·4169
·3655	1·3933	·2720	1·4189
·3513	1·3959	·2618	1·4243

Fig. 2.



Magneto-Optical Dispersion.

Active Isoamyl Alcohol.

The experimental results for the magneto-optical dispersion are given in Table II. (a) and fig. 2.

As previously mentioned, the isoamyl alcohol employed in these investigations was optically active, but, as Perkin* has measured the rotations of both active and inactive isoamyl alcohols for sodium light and found very little difference, it was considered of interest to measure the rotation of the active liquid in the ultra-violet region of the spectrum. It is hoped that later it will be possible to carry out similar investigations with inactive isoamyl alcohol.

Measurements of the natural rotation of a column 17.5 cm. long of the active isoamyl alcohol for sodium light were

TABLE II. (a).

Temperature in degrees Cent.	Wave-length in microns.	Verdet's constant in mins. per cm. gauss.
15.0	.4677	.0213 ₅
15.0	.4606	.0221 ₁
15.0	.4539	.0228 ₇
15.0	.4527	.0230 ₁
15.0	.4386	.0247 ₃
15.0	.4284	.0260 ₉
15.0	.4166	.0278 ₂
15.0	.4072	.0293 ₀
15.0	.3972	.0310 ₆
15.0	.3894	.0325 ₂
15.0	.3811	.0342 ₈
15.0	.3739	.0358 ₃
15.0	.3676	.0373 ₁
15.0	.3612	.0389 ₅
15.0	.3554	.0405 ₆
15.0	.3496	.0421 ₄

carried out with the ordinary laboratory half-shadow polarimeter, and the mean value obtained at 19.5°C. was $-2^{\circ}44'$. The calculated rotations for columns 20 and 30.5 cm. long are $-3^{\circ}7.4'$ and $-4^{\circ}45.8'$ respectively. The natural rotation obtained by Perkin for a column 20 cm. long of his specimen of active isoamyl alcohol was -2.27° at 15°C.

Table II. (b) gives a series of values of n , the refractive index, and δ , Verdet's constant, for isoamyl alcohol taken from fig. 1 and Table II. (a) respectively. From these

* Journ. Chem. Soc. xlv. pp. 469-471 (1884).

values it was found that the experimental results could be represented by the equation

$$\phi = n\delta\lambda^2 = K_1 \left[\frac{\lambda^2}{\lambda^2 - \lambda_1^2} \right]^2.$$

TABLE II. (b).

	λ .	n .	δ .
(a).....	·4386	1·4180	·0247 ₃
(b).....	·4166	1·4203	·0278 ₂
(c).....	·3676	1·4272	·0373 ₄
(d).....	·3496	1·4307	·0421 ₈

From (a) and (b) $\lambda_1 = \cdot 1162 \mu$ and $K_1 = 5\cdot 83_2 \times 10^{-3}$.

„ (a) „ (c) $\lambda_1 = \cdot 1165 \mu$ „ $K_1 = 5\cdot 82_8 \times 10^{-3}$.

„ (b) „ (d) $\lambda_1 = \cdot 1166 \mu$ „ $K_1 = 5\cdot 82_4 \times 10^{-3}$.

„ (a) „ (d) $\lambda_1 = \cdot 1166 \mu$ „ $K_1 = 5\cdot 82_6 \times 10^{-3}$.

The mean values of λ_1 and K_1 are $\cdot 1165 \mu$ and $5\cdot 82_8 \times 10^{-3}$ respectively, and the equation representing the magneto-optical dispersion of active isoamyl alcohol over the range of spectrum investigated is

$$n\delta = 5\cdot 82_8 \times 10^{-3} \frac{\lambda^2}{[\lambda^2 - (\cdot 1165)^2]^2} \cdot \cdot \cdot \quad (A)$$

TABLE II. (c).

	δ (calculated) at 15° C.	δ (observed) at 15° C.
·5893 μ	·0129 ₀	
·4606 μ	·0221 ₄	·0221 ₁
·4527 μ	·0230 ₂	·0230 ₁
·4284 μ	·0260 ₉	·0260 ₉
·3811 μ	·0342 ₆	·0342 ₃
·3612 μ	·0389 ₄	·0389 ₅
·3554 μ	·0405 ₁	·0405 ₆

With values of n read off from fig. 1, δ was calculated by equation (A) for a few wave-lengths at which experimental determinations had been carried out. The observed and calculated values of δ are given in Table II. (c).

Lowry and Dickson* have determined the values of $\frac{\delta_\lambda}{\delta(5461\mu)}$ for active isoamyl alcohol at three wave-lengths in the visible spectrum, and a comparison of their results with those calculated from equation (A) is given in Table II. (d).

TABLE II. (d).

λ .	Magnetic rotary power relative to that at 5461 μ .	
	Lowry & Dickson.	Present results.
•6708 μ	•647	•644
•5893 μ	•850	•848
•4359 μ	1•641	1•650

TABLE III. (a).

Temp. (° C.).	λ (in microns).	δ (min./cm. gauss).	Temp. (° C.).	λ (in microns).	δ (min./cm. gauss).
20•0	•4646	•0183 ₂	20•0	•3805	•0288 ₂
20•1	•4600	•0187 ₃	20•1	•3768	•0294 ₇
20•0	•4587	•0188 ₂	20•0	•3716	•0304 ₇
20•0	•4550	•0192 ₁	20•0	•3693	•0308 ₃
20•0	•4471	•0200 ₁	20•0	•3675	•0312 ₃
20•0	•4397	•0207 ₃	20•0	•3654	•0316 ₃
20•0	•4390	•0208 ₁	20•0	•3639	•0320 ₅
20•0	•4317	•0216 ₂	20•0	•3596	•0329 ₁
20•0	•4260	•0222 ₂	20•0	•3559	•0336 ₃
20•0	•4200	•0229 ₃	20•0	•3550	•0338 ₀
20•2	•4182	•0232 ₁	20•0	•3542	•0340 ₃
20•0	•4121	•0240 ₁	20•0	•3486	•0353 ₆
20•0	•4090	•0243 ₃	20•0	•3463	•0359 ₆
20•0	•4072	•0246 ₆	20•0	3394	•0377 ₉
20•0	•4004	•0256 ₂	20•0	•3366	•0384 ₃
20•0	•3926	•0268 ₂	20•0	•3303	•0401 ₉
20•0	•3894	•0273 ₀	20•0	•3294	•0405 ₂
20•0	•3850	•0280 ₄	20•2	•3245	•0420 ₄
20•0	•3827	•0284 ₆			

According to Perkin's† experiments the specific rotation of isoamyl alcohol (active) at a mean temperature of 19•9° C.

* 'International Critical Tables,' vi. p. 434.

† *Loc. cit.* p. 470.

is 0.9888, and taking .0131 as the value of Verdet's constant for water, the value calculated for isoamyl alcohol at 19.9°C. is .01295. This is in fairly good agreement with the value of Verdet's constant at 15°C. (.0129₀) calculated from equation (A).

Normal Propyl Formate.

The magneto-optical dispersion of normal propyl formate has been investigated from .4646 μ to .3245 μ , and the experimental results are collected in Table III. (a) and fig. 2.

In Table III. (b) values of Verdet's constants and refractive indices corresponding to certain wave-lengths are given, and these values were utilized to calculate the constants of the equation

$$\phi = n\delta\lambda^2 = K_1 \left[\frac{\lambda^2}{\lambda^2 - \lambda_1^2} \right]^2$$

for normal propyl formate.

TABLE III. (b).

	λ .	n .	δ .
(a)4397	1.3848	.0207 ₃
(b)3768	1.3917	.0294 ₇
(c)3596	1.3944	.0329 ₁
(d)3366	1.3989	.0384 ₃
(e)3245	1.4017	.0420 ₄

The values of the function ϕ were calculated for the above wave-lengths, and from several pairs of these values a set of values of λ_1 and K_1 was obtained.

From (a) and (b) $\lambda_1 = .1091 \mu$ and $K_1 = 4.88_8 \times 10^{-3}$.

„ (b) „ (d) $\lambda_1 = .1071 \mu$ „ $K_1 = 4.92_0 \times 10^{-3}$.

„ (c) „ (e) $\lambda_1 = .1068 \mu$ „ $K_1 = 4.93_2 \times 10^{-3}$.

„ (a) „ (d) $\lambda_1 = .1080 \mu$ „ $K_1 = 4.90_2 \times 10^{-3}$.

The mean values of λ_1 and K_1 are .1077 μ and 4.910×10^{-3} respectively, and the magneto-optical dispersion of normal propyl formate over the range of spectrum investigated is represented by the equation

$$n\delta = 4.910 \times 10^{-3} \frac{\lambda^2}{[\lambda^2 - (.1077)^2]^2} \quad (B)$$

A comparison of the results calculated from equation (B) and those obtained experimentally is given in Table III. (c).

Perkin* found the specific rotation of normal propyl formate to be 0.8360 at 23.4°C., and, assuming the value of Verdet's constant for water to be .0131, the value calculated for normal propyl alcohol at 23.4°C. is .01095. This value is in good agreement with that calculated from equation (B) above.

TABLE III. (c).

λ .	δ (observed) at 20°C.	δ (calculated) at 20°C.	λ .	δ (observed) at 20°C.	δ (calculated) at 20°C.
.5893		.0109 ₉	.4004	.0256 ₂	.0256 ₂
.4646	.0183 ₂	.0183 ₉	.3805	.0288 ₂	.0288 ₁
.4550	.0192 ₁	.0192 ₂	.3542	.0340 ₃	.0340 ₄
.4200	.0229 ₃	.0229 ₃	.3294	.0405 ₂	.0405 ₁

The Natural Dispersion Equations and the Values of e/m .

The experiments on magnetic rotation have shown that the magneto-optical dispersion of active isoamyl alcohol and normal propyl formate can be explained by the presence of only one absorption band. Although the refractive indices have not been determined with a high degree of accuracy, calculations show that the natural dispersion can be approximately represented by the equation

$$n^2 - 1 = b_0 + \frac{b_1}{\lambda^2 - \lambda_1^2},$$

where λ_1 has the same value as that determined from the magneto-optical experiments. The values of the constants b_0 and b_1 can be determined from two values of the refractive indices n_x and n_y corresponding to wave-lengths λ_x and λ_y .

Isoamyl Alcohol.

The pairs of values of refractive indices and wave-lengths employed to calculate b_0 and b_1 are given in Table IV. (a).

TABLE IV. (a).

(A)	λ (cm. $\times 10^5$).	n .	(B)	λ (cm. $\times 10^5$).	n .
	6.678	1.407 ₂		5.016	1.413 ₅
	4.526	1.417 ₁		3.879	1.424 ₂
(C)	λ (cm. $\times 10^5$).	n .			
	4.472	1.417 ₅			
	3.367	1.433 ₄			

* Journ. Chem. Soc. i. p. 491 (1884).

1230 Mr. E. Thomas and Prof. E. J. Evans on the

From (A) $b_1 = .959_1 \times 10^{-10}$ and $b_0 = .9580_3$.

„ (B) $b_1 = .978_5 \times 10^{-10}$ „ $b_0 = .9568_7$.

„ (C) $b_1 = .968_7 \times 10^{-10}$ „ $b_0 = .9575_7$.

The mean values of b_1 and b_0 are $.9687 \times 10^{-10}$ and $.9574_9$ respectively, and the natural dispersion of active isoamyl alcohol is represented by the equation

$$n^2 = 1.9574_9 + \frac{.968_7 \times 10^{-10}}{\lambda^2 - (1.165 \times 10^{-5})^2}.$$

A comparison of the results calculated from this equation and the observed values is given below :—

λ .	n (observed).	n (calculated).
.5876	1.409 ₈	1.409 ₅
.4713	1.415 ₇	1.415 ₆
.4275	1.419 ₂	1.419 ₄
.3722	1.426 ₄	1.426 ₅

Normal Propyl Formate.

The pairs of values of n and λ employed in calculating the values of b_1 and b_0 for propyl formate are given in Table IV. (b).

TABLE IV. (b).

(A)	λ (cm. $\times 10^5$).	n .	(B)	λ (cm. $\times 10^5$).	n .
	6.678	1.3747		5.016	1.3808
	4.063	1.3882		3.261	1.4013
(C)					
	λ (cm. $\times 10^5$).	n .			
	4.472	1.3845			
	2.618	1.4243			

From (A) $b_1 = .885_8 \times 10^{-10}$ and $b_0 = .8694_2$.

„ (B) $b_1 = .892_7 \times 10^{-10}$ „ $b_0 = .8694_1$.

„ (C) $b_1 = .912_3 \times 10^{-10}$ „ $b_0 = .8684_2$.

The mean values of b_1 and b_0 are $.896_8 \times 10^{-10}$ and $.8690_8$ respectively, and the natural dispersion of normal propyl formate is represented by the equation

$$n^2 = 1.8690_8 + \frac{.896_8 \times 10^{-10}}{\lambda^2 - (1.077 \times 10^{-5})^2}.$$

A comparison of the results calculated from the above equation with those observed is given in the following table :—

λ .	n (observed).	n (calculated).
·5876	1·3771	1·3773
·4713	1·3827	1·3827
·3655	1·3933	1·3937
·3007	1·4080	1·4081
·2824	1·4146	1·4145

VALUES OF e/m .

The values of e/m for isoamyl alcohol and normal propyl formate were calculated from the equation

$$\frac{e}{m} = - \frac{2K_1 C^2}{b_1},$$

and found to be $1\cdot05 \times 10^7$ and $0\cdot96 \times 10^7$ electromagnetic units respectively.

The values of e/m obtained for isobutyl alcohol* and ethyl formate† were $1\cdot11 \times 10^7$ and $\cdot996 \times 10^7$ respectively. A comparison of the values with those obtained in the present investigation shows that the value of e/m is smaller the greater the molecular weight of the compound of similar type.

In a previous paper‡ it was pointed out that in the case of all the alcohols examined up to that time the values of λ_1 and K_1 increase as the molecular weight of the alcohol increases, and the position of the absorption band is displaced on the average about $\cdot0016 \mu$ towards the red by the addition of CH_2 to the molecular weight. The results obtained in the present investigation for isoamyl alcohol appear to support the above results, as the position of the absorption band is displaced $\cdot0015 \mu$ towards the red (*i. e.*, from $\cdot1150$ to $\cdot1165 \mu$) as compared with isobutyl alcohol, and the value of K_1 ($5\cdot828 \times 10^{-3}$) for isoamyl alcohol is greater than the corresponding value ($5\cdot69 \times 10^{-3}$) for isobutyl alcohol. The present results were, however, obtained with active isoamyl alcohol, but there are reasons for believing that the value of λ_2 will not be materially altered when similar experiments

* Stephens and Evans, *loc. cit.*

† Jenkins and Evans, *loc. cit.*

‡ Stephens and Evans, *loc. cit.*

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are carried out with inactive isoamyl alcohol. A comparison of the results obtained with ethyl and propyl formates also shows that the position of the absorption band is shifted towards the red (from $\cdot 1061\mu$ to $\cdot 1077\mu$) with increase of molecular weight, and that the value of K_1 for propyl formate ($4\cdot 910 \times 10^{-3}$) is greater than the corresponding value ($4\cdot 72 \times 10^{-3}$) for ethyl formate.

SUMMARY.

(a) The magneto-optical and natural dispersion of (active) isoamyl alcohol and normal propyl formate have been investigated for various wave-lengths in the visible and ultra-violet regions of the spectrum.

(b) The magneto-optical dispersion of isoamyl alcohol and normal propyl formate for the ranges of wave-length investigated can be represented by the equations

$$n\delta = 5\cdot 82_8 \times 10^{-3} \frac{\lambda^2}{[\lambda^2 - (\cdot 1165)^2]^2}$$

and

$$n\delta = 4\cdot 910 \times 10^{-3} \frac{\lambda^2}{[\lambda^2 - (\cdot 1077)^2]^2}$$

respectively, where n is the refractive index and δ Verdet's constant in minutes per cm. gauss for wave-length λ .

(c) The natural dispersion of isoamyl alcohol and normal propyl formate can be represented by the equations

$$n^2 = 1\cdot 9574_9 + \frac{\cdot 968_7 \times 10^{-10}}{\lambda^2 - (1\cdot 165 \times 10^{-5})^2}$$

and

$$n^2 = 1\cdot 8690_8 + \frac{\cdot 896_8 \times 10^{-10}}{\lambda^2 - (1\cdot 077 \times 10^{-5})^2}$$

respectively.

(d) The values of e/m for isoamyl alcohol and normal propyl formate calculated from the magneto-optical and natural dispersion measurements are $1\cdot 05 \times 10^7$ and $0\cdot 96 \times 10^7$ e.m.u. respectively.

November 1930.

CV. *The Warming of a Room.* By A. F. DUFTON, M.A.,
D.I.C., the Building Research Station *.

[Plate XV.]

1. **I**N a previous communication dealing with the influence of the fabric of a wall †, reference was made to the well-known fact that some rooms are not readily warmed; it was pointed out that although complete analysis of the warming of a room is not easy it is clear that a room with masonry walls is much more readily warmed if lined with wood or other insulation of small thermal capacity. The problem is one of some importance and its experimental investigation has been hampered by the lack of suitable accommodation.

Although accommodation may perhaps be forthcoming at the Building Research Station during the next year, it now appears probable that the investigation will be further delayed and the facilities devoted to more urgent work. In these circumstances an account of some experimental observations of a somewhat limited scope, made in a private house, are perhaps worthy of record.

2. A plan of the small furnished room in which the experiments were made is shown in fig. 1. The room, which is 9 feet in height, is on the ground floor and there is no room above it. The ceiling is of lath and plaster and is not boarded above the joists.

3. In a paper on "The Effective Temperature of a Warmed Room" ‡, the difficulty of specifying the temperature of a room with air and walls at different degrees was pointed out, and a physical instrument, the eupatheoscope, was described which is designed to measure and record a quantity analogous to the physiological sensation induced by a cool environment. The instrument is essentially a black-painted hollow copper cylinder at 75° F., the loss of heat from which is recorded in degrees of effective temperature, the effective temperature being defined as that temperature of a uniform enclosure in which, in still air, a sizable black body at 75° would lose heat at the same rate as in the environment.

4. The room was warmed by means of a gas-fire burning 0·2 therm per hour and records were taken with a eupatheoscope 8 feet from the fire. The room was then lined

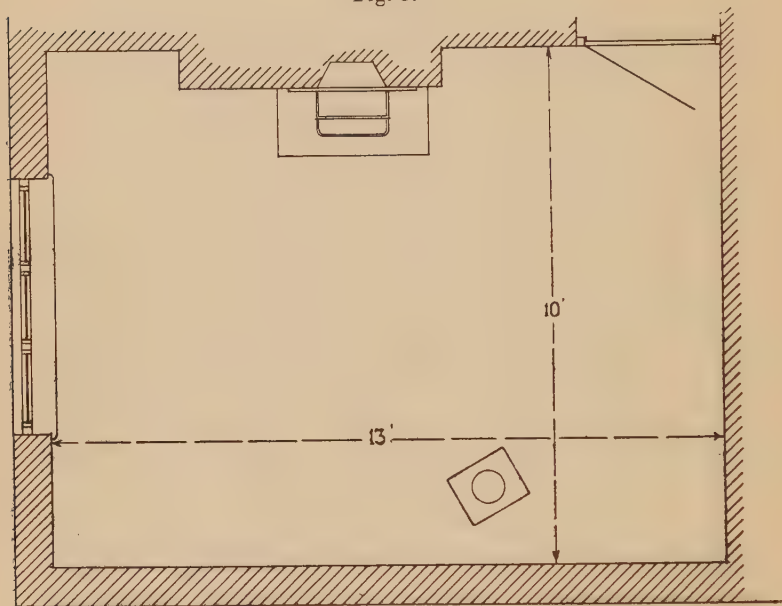
* Communicated by R. E. Stradling.

†. *Phil. Mag.* iv. p. 888 (1927).

‡ *Phil. Mag.* ix. p. 858 (1930).

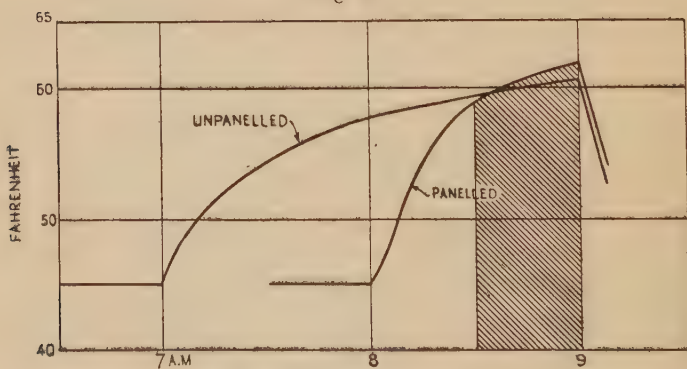
with oak panelling as in Pl. XV. and further records were taken. It was observed that the panelled room was warmer

Fig. 1.



Plan of Room.

Fig. 2.



Room used for half an hour.

after half an hour than the unpanelled room after an hour and a quarter.

The records of effective temperature shown in fig. 2 illustrate that if the room is required for a period of half an hour the fire does not need to be lit so early now that the room is panelled and that in this experiment the same degree of warmth was obtained with half the fuel consumption of the unpanelled room.

It may perhaps be emphasized that the gas-fire does not attain its full intensity during the first quarter of an hour and that, without taking furniture into account, only one half of the area of the boundary surfaces of the room was changed by the panelling.

Records taken with the eupatheoscope screened from the fire confirmed that the panelled room was warmer after half an hour than the unpanelled room after an hour and a quarter.

5. In a room used for dining, a gas-fire tends to give an unsuitable distribution of warmth. It is regretted that it has not been possible to extend the experiments to include observations upon some form of air-heating, which in a panelled room would probably provide a rapid and uniform warmth.

CVI. *Flow of Water through a Circular Tube with a Central Core and through Rectangular Tubes.* By Professor F. C. LEA, D.Sc., M.Inst.C.E., and A. G. TADROS, Ph.D.*

THE general expression for the velocity of any streamline at any radius r in a circular tube with or without a central core is

$$u = -\frac{1}{4\mu} \frac{dp}{dz} r^2 + A \log r + B; \quad \dots \quad (1)$$

$\frac{dp}{dz}$ is negative, μ is the coefficient of viscosity.

The constants A and B can be found assuming no slip at the boundaries. When there is no core and the tube has the radius a_1

$$u = -\frac{1}{4\mu} \frac{dp}{dz} (a_1^2 - r^2), \quad \dots \quad (2)$$

and
$$\frac{du}{dr} = \frac{r}{2\mu} \frac{dp}{dz} \quad \dots \quad (3)$$

* Communicated by the Authors.

The flow per unit time is

$$Q = \frac{\pi a_1^4}{8} \frac{dp}{dz}, \quad \dots \dots \dots (4)$$

and the stress R at the boundary is

$$R = \mu \frac{du}{dr} = \frac{a_1}{2} \frac{dp}{dz} \cdot \dots \dots \dots (5)$$

When there is a core of radius a_0

$$A = \frac{1}{4\mu} \frac{dp}{dz} \left(\frac{a_0^2 - a_1^2}{\log \frac{a_0}{a_1}} \right), \quad \dots \dots \dots (6)$$

$$B = \frac{1}{4\mu} \frac{dp}{dz} \left(\frac{a_0^2 \log a_1 - a_1^2 \log a_0}{\log \frac{a_0}{a_1}} \right), \dots \dots \dots (7)$$

$$u = -\frac{1}{4\mu} \frac{dp}{dz} \left\{ r^2 + \frac{a_1^2 - a_0^2}{\log \frac{a_0}{a_1}} \log r + \frac{a_0^2 \log a_1 - a_1^2 \log a_0}{\log \frac{a_0}{a_1}} \right\}, \dots \dots \dots (8)$$

$$\frac{du}{dr} = -\frac{1}{4\mu} \frac{dp}{dz} \left(2r - \frac{a_1^2 - a_0^2}{r \log \frac{a_1}{a_0}} \right) \cdot \dots \dots \dots (9)$$

The stress at any radius r is

$$R_r = -\frac{1}{4\mu} \frac{dp}{dz} \left\{ 2r - \frac{a_1^2 - a_0^2}{r \log \frac{a_1}{a_0}} \right\} \cdot \dots \dots \dots (10)$$

The flow per unit time is

$$\begin{aligned} Q &= -\frac{\pi}{8\mu} \frac{dp}{dz} \left\{ a_1^4 - a_0^4 - \frac{(a_1^2 - a_0^2)^2}{\log \frac{a_1}{a_0}} \right\} \cdot \dots \dots (11) \\ &= -C \frac{\pi}{8\mu} \frac{dp}{dz}, \end{aligned}$$

in which C clearly depends upon the dimensions, or

$$Q = Q_1 - K \left\{ a_0^4 + \frac{(a_1^2 - a_0^2)^2}{\log \frac{a_1}{a_0}} \right\},$$

where Q_1 is the flow without the core and K is $\frac{\pi}{8\mu} \frac{dp}{dz}$.

Table I. shows the percentage reductions of flow as compared with flow through a tube without a core for ratios of a_1 , the radius of the outer tube, to a_0 , the radius of the core, of from 2 to 1000.

TABLE I.
Tube of radius a_1 with core of radius a_0 .

Ratio $\frac{a_0}{a_1}$	2	4	5	10	20	50	100	1000
Percentage of reduction of flow as compared with a tube without a core.	87.4	63.8	57.4	42.6	33.2	25.5	22	14.5

Fig. 1.

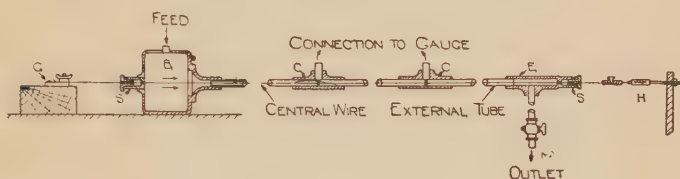
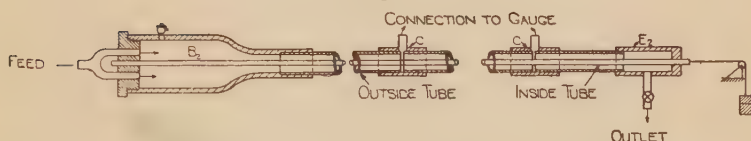


Fig. 2.



It seemed difficult to imagine that a wire only $1/1000$ of the diameter of the tube could diminish the flow by 14.5 per cent., and it appeared of interest to test the assumptions experimentally. If experiment indicated that such a small wire did not reduce the percentage of flow by the estimated amount shown in Table I. then, either the assumption of no slip at the convex boundary is invalid, or else some other explanation of the discrepancy must be sought, and it appeared that the experiments might throw light upon the probable condition of flow near to boundaries.

Experimental Results.

Experiments were carried out with the apparatus shown in figs. 1 and 2. A constant head was maintained from a tank which could be placed at various heights. Wires of various diameters were placed concentric with the tube and

stretched by a load as shown. The logarithmic plottings of $\frac{dp}{dz}$ and v , the mean velocity, are shown in fig. 3.

Fig. 3.

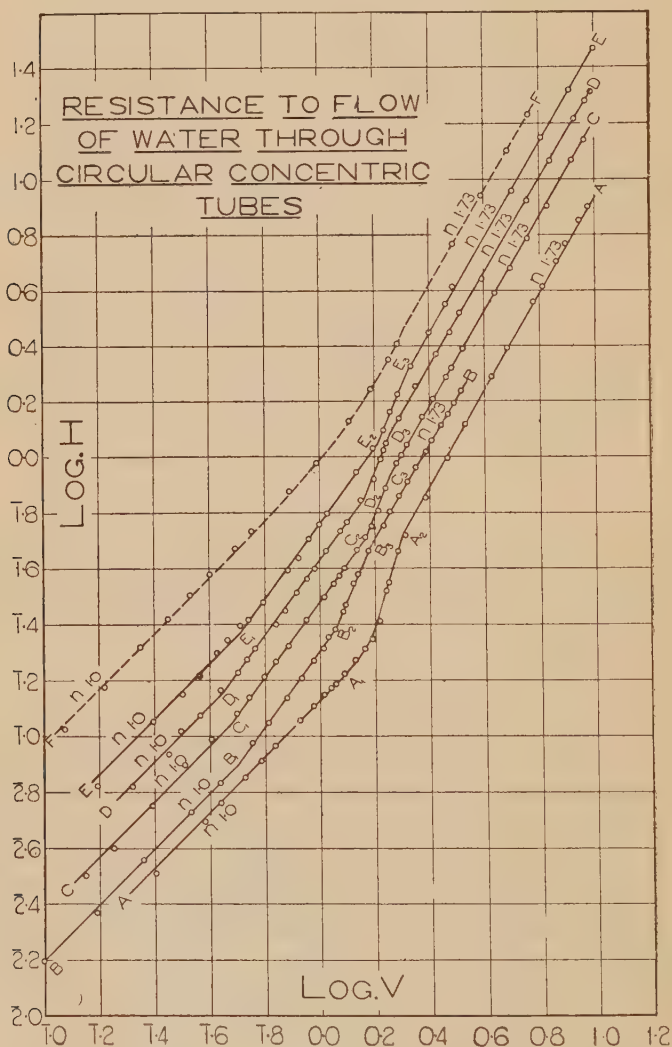


Table II. shows, for the same values of $\frac{dp}{dz}$, the theoretical percentage reductions of flow, assuming no slip at the two

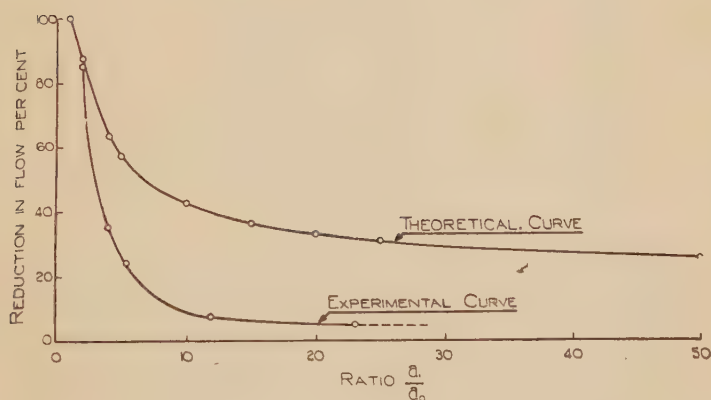
boundaries, and the percentage reduction of flow determined experimentally.

TABLE II.

		<i>a.</i>	<i>b.</i>	<i>c.</i>	<i>d.</i>	<i>e.</i>	<i>f.</i>
Internal radius of tube, cm. }	a_1	0.3219	0.3219	0.3219	0.3219	0.3219	0.6317
External radius of core, cm. }	a_0	No core	0.0140	0.0272	0.0597	0.0380	0.3150
Ratio	23.04	11.84	5.38	3.89	2.02
Percentage reduction of flow as compared with tube without core. }	Theoretical }	Nil	31.60	39.60	55	64.6	87.4
	Experimental }	Nil	5	7.5	24	35	85.4

The results are shown plotted in fig. 4. The curves indicate

Fig. 4.



that when $\frac{a_1}{a_0}$ approaches 2 the theoretical and experimental discharges practically agree.

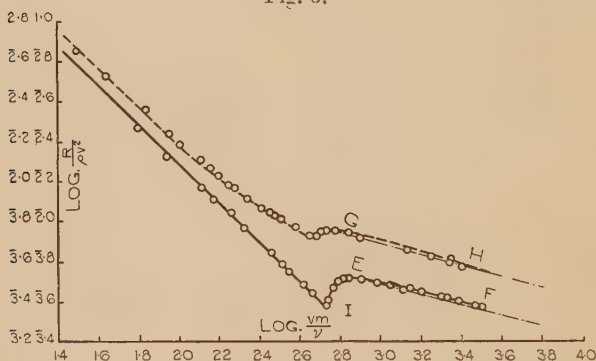
No confirmation of these tests has been found, but Mallock and Fleure* have obtained comparable results from experiments on viscous fluids between rotating coaxial cylinders.

* Phil. Trans. Roy. Soc. p. 41 (1896).

That the central core affects the flow is shown clearly by the log plottings of $\frac{dp}{dz}$ and v (fig. 3). Plots A and F refer to the two brass tubes of 0.3219 and 0.6317 cm. radius respectively, the latter having a core of 0.3150 cm. radius. The lower critical velocity in the smaller tube is at A_1 . With the cores in place in the tube the "lower" critical velocities occur much earlier, *i. e.*, at B_1 , C_1 , D_1 , and E_1 .

The effect is also shown in fig. 5, in which $\log \frac{R}{\rho v^2}$ is plotted against the log of Reynolds' number $\frac{vm}{\nu}$, where m is the

Fig. 5.



hydraulic mean depth. The two curves are displaced vertically for clearness. For round tubes the point I at which instability occurs is well defined, but for the tubes with a central core some breakdown of stream-line flow clearly occurs much earlier. In the turbulent regions EF and GH the points lie in both cases very near to the curves which have been drawn to represent the results obtained by Stanton and Pannell* for brass tubes. That the condition of flow in the tube with the core is different from that in tubes of other shapes than circular is seen by comparing fig. 5 with that of fig. 6, obtained from experiments on the flow in rectangular and square tubes.

It will be seen that the plotted points of fig. 6 agree well with those obtained by Cornish† for rectangular tubes and

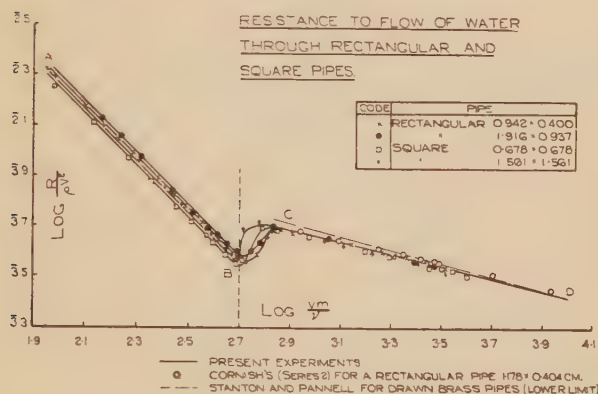
* Phil. Trans. A. vol. ccxiv.

† Proc. Roy. Soc. A (1927).

by Stanton for circular tubes. The form of the external boundary can be considerably changed without affecting the stream-line flow, and, further, the discharge stream-line flow agrees with the results of calculations assuming no slip at the boundary (see Appendix I.).

The explanation of the difference of the flow given by equation (11) and experiment appears to necessitate either definite slip at the boundary of the core or else a much more rapid rate of change of velocity at this boundary than would be the case if there were no slip.

Fig. 6.



Let it first be assumed that slip takes place at the core, and that the velocity $u_0 = \lambda v$, where v is the mean velocity. From the general equation (1), and writing

$$K = \frac{1}{4\mu} \frac{dp}{dz},$$

$$A = \frac{K(a_1^2 - a_0^2) + \lambda v}{\log \frac{a_0}{a_1}},$$

and

$$B = \frac{K(a_0^2 \log a_1 - a_1^2 \log a_0) - \lambda v \log a_1}{\log \frac{a_0}{a_1}};$$

the velocity at radius r is

$$u = K \left[r^2 + \frac{a_1^2 - a_0^2}{\log \frac{a_0}{a_1}} \log r + \frac{a_0^2 \log a_1 - a_1^2 \log a_0}{\log \frac{a_0}{a_1}} \right] + \lambda v \frac{\log r - \log a_1}{\log \frac{a_0}{a_1}}, \quad \dots \quad (12)$$

and

$$Q = Q_0 + \pi K \lambda v \left(\frac{a_1^2 - a_0^2 - 2a_0^2 \log \frac{a_1}{a_0}}{2 \log \frac{a_1}{a_0}} \right), \quad \dots \quad (13)$$

$$\frac{du}{dr} = K \left(2r - \frac{a_1^2 - a_0^2}{r \log \frac{a_1}{a_0}} \right) - \frac{K \lambda v}{r \log \frac{a_1}{a_0}} \dots \dots \dots (14)$$

Fig. 7.

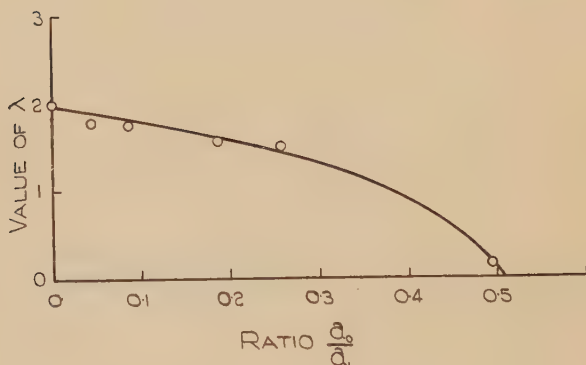
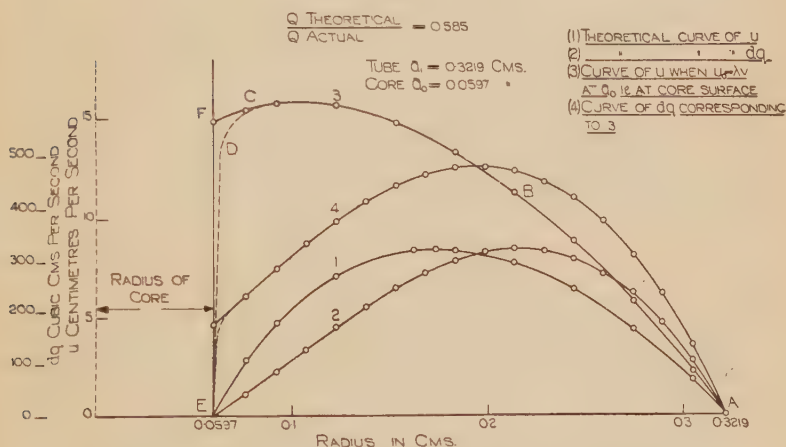


Fig. 7 shows values of λ , determined by equating the experimental values of Q to equation (14). This curve, if continued, cuts the axis at $\frac{a_0}{a_1}$ nearly equal to 0.5, and suggests that for values of $\frac{a_0}{a_1} > 0.5$ the flow is stream line and that there is no slip at either boundary. This assumption of definite slip at the surface of small cores, although appearing to agree with the results of the tests, is, however, by no means satisfactory.

Reference has already been made to the early breakdown of stream-line flow in the cored tubes, as evidenced by the log plottings of figs. 3 and 5. These indicate that at small velocities some turbulent motion is set up which does not persist throughout the mass.

In fig. 8 is shown plotted a curve of velocity, no. 1, when $a_1=0.3219$ cm. and $a_0=0.0597$ cm., determined from equation (8). Curve no. 2 shows the rate of discharge at any radius on the assumption of no slip at the boundary. The volume of the solid of revolution from this curve is proportional to the theoretical discharge. Curves nos. 3 and 4 show the velocity and discharge curves, respectively, on the assumption of a

Fig. 8.



slip velocity $u_0=\lambda v$ at the core boundary. The volume of the solid of revolution of this curve is equal to the discharge obtained experimentally.

It will be seen from curve no. 3 that the value of $\frac{du}{dr}$, or the slope of the tangent at F, is less than at the wall of the outer tube, and if the stress $R=\mu \frac{du}{dr}$ is a criterion of breakdown of the stream-line flow, then, with the velocity curve no. 3 an earlier breakdown of the stream-line flow with the core in place would not be expected. The stress per unit area at the boundary of the outer tube at the lower critical velocity (curve A, fig. 3) was 7.5 dynes per square cm. The

stress at the core boundary at the lower critical velocity D (fig. 3), as determined by equation (10), was only about 4 dynes per square cm.

If Reynolds' number at the lower critical velocity be taken as 2000, and this be assumed equal to $\frac{vd}{\nu}$, d being equal to $2a_0$, then $v = 237$ cm. per second. The experimental value for v at 6.7°C . was only 13.3 cm. per second. The value of $\frac{du}{dr}$ corresponding to 237 cm. per second is about $17,000$.

If equation (1) be solved on the assumption that the slope of the velocity curve at the core boundary has some positive value γa_0 , the velocity u at radius r is

$$u = K \left\{ r^2 - a_0^2 \left(\frac{\gamma}{K} - 2 \right) \log \frac{a_1}{r} - a_1^2 \right\}, \quad \dots \quad (15)$$

and

$$\frac{du}{dr} = K \left\{ 2r + a_0^2 \left(\frac{\gamma}{K} - 2 \right) \frac{1}{r} \right\}. \quad \dots \quad (16)$$

If Q is the experimental discharge

$$Q = \int_{a_0}^{a_1} 2\pi r dr \cdot u.$$

Inserting experimental values of K at the lower critical velocity indicates that equations (15) and (16) are impossible and that very near to the core boundary equation (1) does not hold.

The difference of curves 1 and 3 (fig. 8) and of curves 2 and 4 can be explained by assuming that near to the core boundary there is a thin film in which the slope is above the critical value, but at some small distance ∂r from the core the velocity is given very approximately by equation (12).

That near to the core boundary early disturbance takes place which cannot cause complete turbulent flow is also indicated

by the $\log \frac{dp}{dz}$, v curves, fig. 3. For a considerable range after the clear indication of the breaking-up of the streamline flow the points lie on a straight line having a slope of about 1.36 , or, in this region,

$$\frac{R}{\rho v^2} = A \left(\frac{\nu}{vm} \right)^{.64}, \quad \dots \quad (17)$$

where m is the hydraulic mean depth.

In the turbulent area

$$\frac{R}{\rho v^2} = A_1 \left(\frac{v}{vm} \right)^{.27} \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and in the stream-line area

$$\frac{R}{\rho v^2} = 2 \left(\frac{v}{vm} \right) \quad . \quad . \quad . \quad . \quad . \quad (19)$$

The condition of flow represented by equation (17) persists in the cases C, D, and E, fig. 3, as far as the lower critical velocity A_1 for the tube A. The marked difference between the curve F and the other curves of fig. 3 is clearly seen. The curve F shows the plottings for a tube with a core having a radius equal to half that of the tube radius, in which the discharge is only slightly less than the theoretical discharge and there is much less evidence of disturbance at the convex boundary.

When the turbulent motion is persistent in a tube there is a layer, as shown by Stanton and as indicated by Hele-Shaw and other earlier workers, which adheres to the boundary. At turbulent velocities it would appear that, as Karman* suggested, the velocity distribution adjoining the stream lines at the boundary is given by

$$u \propto r^n \quad . \quad . \quad . \quad . \quad . \quad (20)$$

The index,

$$n = \frac{k}{2-k}.$$

and the resistance per unit area is

$$R = A \rho v^2 \left(\frac{v}{vd} \right)^k.$$

Such a condition would appear to obtain near to the core at velocities well below those at which the motion becomes turbulent, but the disturbance produced by the core does not persist, except for a very small distance from the core, until the slope of the velocity curve at the tube boundary becomes such as to cause the formation of partial turbulence at this boundary.

In fig. 8, instead of the curve 3 cutting the core boundary at F there is a stream-line portion ED joining the curve CA by a curve DC represented by equation (20).

* Gauert, 'Aerofoil and Airscrew Theory,' p. 108.

Summary.

The tests indicate that when a small core is placed at the centre of a circular tube the discharge at velocities corresponding to small Reynolds' numbers is much greater than theory gives on the assumption of no slip at the boundary, and that small turbulent disturbances, which originally existed in the flowing water and that would be destroyed in a circular or rectangular tube without a core up to velocities corresponding to a Reynolds' number of 2000, persist at velocities much lower than in the circular or rectangular tube, and these small disturbances have the same effect as though slip was taking place at the inner core. When the radius of the core is equal to half the radius of the tube the theoretical and experimental values agree. If v is the mean velocity of flow, the apparent velocity of slip at the boundary is given by

$$v = 2.82 \sqrt{0.5 - \frac{a_0}{a_1}},$$

where a_0 is the radius of the core and a_1 of the tube.

APPENDIX.

From theoretical considerations it is not difficult to deduce for rectangular tubes and stream-line flow the following formulæ:—

Mean velocity flow

$$v = \frac{Cb^2}{3\mu} \frac{dp}{dz}.$$

Discharge per unit time

$$Q = \frac{C4ab^3}{3\mu} \frac{dp}{dz},$$

and

$$\frac{R}{\rho v^2} = \frac{3m^2}{Cb^2} \frac{\nu}{v.m},$$

in which

$$C = \left[1 - \frac{0.6274}{r} \left(\tanh \frac{\pi r}{2} + 0.0045 \right) \right],$$

and $r = \frac{a}{b}$, $2a$ and $2b$ being the lengths of the sides of the rectangle.

When

$$a=b, r=1, \text{ and } C=0.4218.$$

Then

$$\frac{R}{\rho v^2} = 1.78 \frac{v}{v \cdot m}.$$

When $a=\infty$, or the flow is between parallel plates of infinite width,

$$\frac{R}{\rho v^2} = \frac{3v}{vm}.$$

In general

$$\frac{R}{\rho v^2} = K \frac{v}{v \cdot m}.$$

Fig. 9.

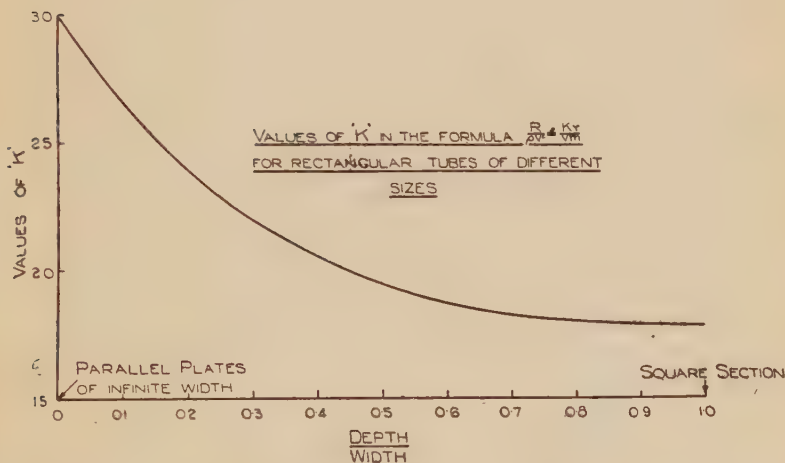


Fig. 9 shows the values of K for the ratio, $\frac{\text{depth}}{\text{width}}$, of rectangular tubes.

Experiments were carried out on tubes shown in Table III. kindly supplied by Messrs. Muntz, of Birmingham.

TABLE III.

$2a.$	$2b.$
0.942 cm.	0.400 cm.
0.196 „	0.937 „
0.678 „	0.678 „
1.561 „	1.561 „

CVII. *The K_{α} -lines of the Light Elements: Remarks on a Paper by M. Söderman.* By ERIK RUDBERG *.

IN the October 1930 issue of the 'Philosophical Magazine,' Mr. Söderman† has given an interesting report on some recent precision measurements of his in the soft X-ray region, using the plane grating method first introduced by Compton and Doan, which has since been so successfully developed to a very high degree of perfection by Siegbahn and his collaborators at Uppsala. There should be no doubt that the values for the K-spectra of the light elements thus secured are more accurate than those obtained by any other of the methods hitherto used.

These methods are briefly reviewed by Söderman in the first sections of his paper, and some of the results which they have yielded are given in Tables I.-IV. and discussed. In Table II. (p. 602) Söderman compared what he purported to be my results‡ with his own, and concluded that "the agreement with the results obtained by the grating method is not very good, Rudberg's values being evidently too low." My results depended on the measurement of the velocities of photoelectrons, and the *final results*, as given on pp. 79, 81, and 84, and also in the summary, p. 144, of my paper§, are the only ones suitable for comparison. Since Söderman overlooked this fact in his criticism of my work I feel that it is desirable to correct this error. *When the comparison is made using my final values, as shown in the following table, it is evident that the agreement between Söderman's results and my own is remarkably good:—*

Element.	4 Be.	5 B.	6 C.
K_{α} (velocity of photoelectrons) volts,	100	183	277
K_{α} (plane grating: Söderman), volts,	106.7 ± 0.14	182.3 ± 0.27	277.2 ± 0.25
Approx. width of the lines, volts ...	9.2	5.4	6.2

In the table, which shows the comparison between my results and the precision measurements of Söderman (Table IX., p. 616), in which he used a plane grating, I have taken the liberty to recalculate Söderman's data for the errors of measurement and the line width from wave-length measure (Å) to the unit (volt) in which the rest of the quantities of this table is expressed. It should be mentioned that the estimated probable error in my measurements was given as ± 10 volts in the original paper.

* Communicated by the Author.

† Phil. Mag. x. p. 600 (1930).

‡ Kungl. Sv. Vetenskapsakad. Handl. ser. 3, vii. no. 1 (1929).

§ Loc. cit.

In my experiments a thin strip or rod was irradiated by soft X-rays, and the velocity of the escaping photoelectrons was determined by finding the magnetic field, applied parallel to the strip, which was just sufficient to prevent the electrons from reaching a cylindrical collecting electrode, coaxial with the strip and (usually) kept at the same potential as the latter. The arrangement was essentially the one described in an earlier paper published in the 'Proceedings of the Royal Society'*. In this way a measure of the kinetic energy with which the electron enters the space between emitter and collector is obtained. It appears that the values of Söderman's Table II. refer to this quantity. To obtain the energy of the radiation quantum, however, it is necessary to consider the work performed by the electron in escaping from the emitter. In my paper therefore I added 4.5 volts to the measured kinetic energy, which was taken to be the work function for the material of the strips (Pt and Cu), thus obtaining the values listed above. Strictly speaking this is only correct if the contact potential difference between the emitter and the collector is negligible. In general there will exist an electric field between the two. When the central electrode is narrow compared with the radius of the collector this field is concentrated to the immediate neighbourhood of the emitter, and the potential may be regarded as constant over the entire space where any deflexion occurs and its value equal to the value at a point immediately outside the surface of the collector†.

The quantity which should be added to the measured kinetic energy to obtain the energy of the radiation quantum is therefore the work function for the collector. This was made of copper, and was covered with a layer of lampblack. The correction applied in my original paper may for this reason be in error by the contact P.D., which for the substances used may have amounted to one-third of a volt‡. The work functions involved cannot, however, be ascertained to this accuracy, and there would be no reason to alter the values previously published.

The great width of these lines found by Söderman is a result of considerable interest. With the high accuracy attained by the grating method it would seem promising to look for a difference in the K-frequencies emitted by gaseous atoms and by the same atoms bound in chemical compounds

* E. Rudberg, Proc. Roy. Soc. A, cxx. p. 385 (1928).

† Cf. loc cit., *Kungl. Sv. Vetenskapsakad. Handl.* p. 45, and Proc. Roy. Soc. p. 398.

‡ Dushman, Int. Crit. Tables, vi. pp. 54, 57 (1929).

or in the solid state. The arrangement of atoms in the surface of the anticathode may also be of importance; thus the soft X-ray frequencies stimulated by more penetrating electrons need not necessarily coincide exactly with those emitted near the critical potentials. With the high input in the anode focal spot in Söderman's tube it does not seem impossible that evaporation could have taken place, and that the radiation studied originated partly in the solid, partly in the gaseous state. If some such circumstances should have played a rôle in connexion with the observed line width it seems possible that effects of the same kind could account for the differences between the wave-length values found by different investigators.

Bartol Research Laboratory,
Swarthmore, Penna.
February 18, 1931.

CVIII. A Galvanometric Method of Measuring an Electrolytic Resistance. By J. A. C. TEEGAN, M.Sc., A.Inst.P.

IN a recent note to 'Nature,' (Oct. 6th, 1930) I described a simple method of eliminating the telephone in the ordinary laboratory bridge method of measuring electrolytic resistance. It consists in making use of the "half-wave rectification" property of an ordinary triode to detect the alternating current flowing in the telephone branch of the bridge. The circuit used is shown in the accompanying figure. The ratio arms A and B consist of two resistances of 50 ohms, C is a sliding resistance (0-150 ohms), and the electrolyte is inserted in the fourth arm. When a telephone is employed a sound is heard on the phones even when the bridge is balanced, owing to the presence of unequal inductances and capacity in the bridge arms; and it is the difficulty of determining the correct position of minimum sound that renders the method so unsatisfactory.

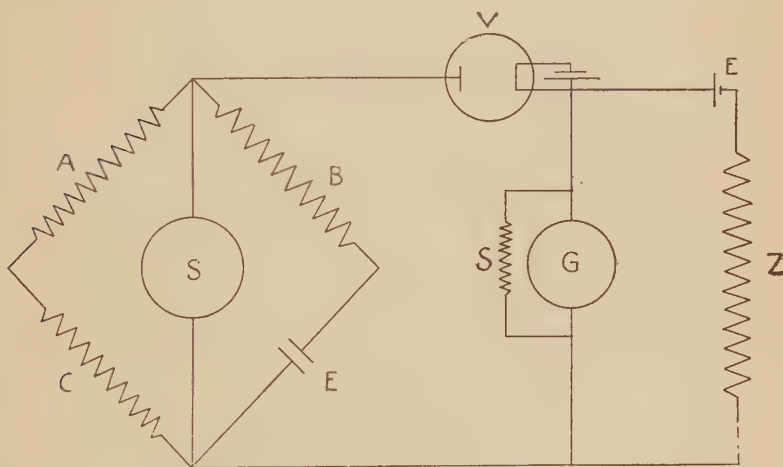
With the galvanometric device described the presence of inductance and capacity is shown by the fact that the galvanometer does not come to zero when the bridge is balanced, but attains a minimum deflexion.

With a sensitive galvanometer this minimum deflexion will be too large. It may be compensated for by the balancing device EX. With the galvanometer shunted X

* Communicated by the Author.

is increased or decreased until the minimum deflexion is eliminated, and with the bridge balanced the galvanometer indicates zero. It should be noted that the galvanometer deflexion is always in the same direction, and does not, as in the case of the D.C. bridge, pass from a positive value through zero to a negative value.

At Rangoon, where these tests were made, a suitable steady low A.C. source was not available, and the ordinary city A.C. source transformed down to 20 volts was employed. The resistances A, B, and C were of high current capacity, to minimize heating disturbances. Using a Pye Unipivot galvanometer high sensitivity was obtainable, and the following set of observations were carried out, to compare



the measurements of the resistance of copper sulphate solutions (at 0°C.), by the telephone and galvanometric methods:—

Electrolyte.	Telephone method.	Galvanometer method.
1.	26.95 ohm.	27.52 ohm.
2.	40.40 „	41.13 „
3.	65.74 „	64.93 „
4.	105.00 „	105.77 „

In no case was the telephone measurement of minimum sound very accurate, and it is likely that the discrepancies in the two sets of measurements are due to inaccuracy in the telephone method.

A trial experiment made with an ordinary "buzzer" as A.C. source, using a galvanometer of sensitivity 20 divisions per microamp., indicated that the sensitivity was about the same as that obtained with the higher voltage, but it was impossible, owing to unsteadiness, to make any sort of accurate measurements. The test indicated, however, that with a suitable steady A.C. source of a few volts the method would appear to have every advantage over the ordinary telephone method.

London,
December 1930.

CIX. *On the Change of the Dielectric Constant
produced by Free Electrons.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN the supplementary February number of this Magazine Mr. W. E. Benham mentions, on p. 490, a paper of mine (*Annalen der Physik*, iii. p. 993, 1929). One of the equations in my paper he applies upon conditions altogether different from those for which it was derived. It is not surprising that he finds discrepancies between my equation, thus treated, and his own. My formula for the dielectric constant of a space filled with electrons was

$$\epsilon = 1 - \frac{4\pi N e^2}{m p^2} \left(1 - \frac{\sin pT}{pT} \right),$$

where N is the density of electrons, $p/2\pi$ the frequency, and T the time during which the electrons remain between the condenser plates. My paper was occasioned by another by Bergmann and Düring (*Annalen der Physik*, i. p. 1041, 1929), and my formulæ were explicitly stated to be valid only under the experimental conditions of that paper, viz., high vacuum. If gas is let into the tube T cannot, as Mr. Benham seems to believe, continue to signify the time that the electrons remain between the condenser plates. In the latter case it must signify the time between two consecutive collisions between an electron and the gas molecules. As this time will have values between 0 and ∞ , distributed according to statistical laws, it is necessary to take the mean value in a manner well known from earlier papers by Salpeter and others. This point will be further investigated in a paper to be published later.

Mr. Benham further criticizes me for overlooking the effect of space charge. The cause is simply that space charge could not have any appreciable influence under the experimental conditions of Bergmann and Düring, conditions totally different from those of Mr. Benham.

Yours faithfully,

SVEN BENNER.

Physical Institute of the
University of Stockholm.
February 26th, 1931.

CX. *Notices respecting New Books.*

Critique of Physics. By L. L. WHYTE. (Kegan Paul, 1930. 10s. 6d.)

"THIS work is written in the belief that the time is ripe for a revision of the fundamental concepts underlying both relativity theory and quantum theory, since this is an essential preliminary to a comprehensive theoretical synthesis, and that the empirical facts required for this task are now available."

Such is the bold aim of the Author of this work.

This is indeed an interesting thesis and one can only congratulate the Author on his courage in attempting, in this essay, to make this aim precise and to suggest a method which can be used in the proposed revision.

This book should be read and studied by all those whose who are interested in the "imaginative" aspect of modern physical theories. Scientific Method, which for so long has been primarily concerned with the simpler problems of induction from data yielded by experiment and observation, has of late entered the rarer atmosphere of scientific theory, where the fundamental objective is the study of the inter-relations of theories and their restatement in terms of ever more powerful concepts. In this domain the material used is of a radically different type. Instead of atomic facts obtained from experience being built into inductions, theories themselves built out of inductions resting on vast numbers of primary facts are welded together to make an ever more comprehensive attack on the problem of our knowledge of the external world.

In this work the Author investigates the present situation. The essay is remarkable for the freshness of its outlook and for its concise presentation of the essential features which distinguish current physical theory from classical theory. With its broad synthetic outlook, it has an originality which we do not remember having encountered in any of the numerous treatises on these subjects which have appeared with such amazing rapidity since relativity and quantum theory became of general interest to all scientific workers. Specially refreshing is the Author's attempt,

bold as it is, to formulate a programme suggesting new lines along which physical thought may advance.

Introduction to Vector Analysis. By L. R. SHORTER. (Macmillan and Co., London, 1931. 8s. 6d.)

THIS is a convenient and well-written introduction to Vector Analysis, particularly useful to students in that it contains many fully worked examples. The Author's aim has been to enable a mathematical student with no previous knowledge of the subject to learn enough to follow the important applications given in more advanced text-books. This book can be strongly recommended.

Analysis Situs. By OSWALD VEBLER. Second edition. (Published in New York by the American Mathematical Society, 1931. 2 dollars.)

THIS well-known book now appears in its second edition in response to the demand for copies of the original issue of 1921, now for some years out of print.

There have been few changes. Chief of these is the inclusion of Professor Veblen's paper on "Intersection Numbers" and of the paper written jointly by Professor Veblen and Dr. Philip Franklin on "Matrices whose Elements are Integers."

We have therefore but to welcome the reappearance of this important volume and recommend it anew to students.

CXI. *Proceedings of Learned Societies.*

GEOLOGICAL SOCIETY.

[Continued from p. 1008.]

April 1st, 1931.—Prof. W. W. Watts, LL.D., Sc.D., F.R.S.,
Vice-President, in the Chair.

THE following communications were read:—

1. 'The Glacier-Lakes of Eskdale, Miterdale, and Wasdale, Cumberland; and the Retreat of the Ice during the "Main Glaciation".' By Bernard Smith, M.A., Sc.D., F.G.S.

The ground and its glacial history, described in this communication, furnishes a link between that discussed by the Author before the Society in 1912 ('The Glaciation of the Black Combe District, Cumberland'), and that investigated by the Author and his colleagues in the Whitehaven and Workington district.

From a review of the available evidence in the north-west of England it is concluded that the Great Ice Age in Cumberland and the Irish Sea basin comprised at least three main episodes. This paper is confined chiefly to the second—that of the 'Main Glaciation'.

The withdrawal of the combined Irish Sea and Lake District ice towards the close of this episode is discussed, and it is concluded that the split between the two ice-sheets progressed northwards, Irish Sea ice tending to shrink on the one hand towards the sea-basin westward and north-westward, whilst the Lake District ice—breaking up into tongues or local glaciers—tended to shrink north-eastward and eastward. Glacier-lakes, impounded between the Irish Sea ice and the snouts of the retiring Lake District valley-glaciers, were accompaniments of this retreat, but formed a consecutive rather than a continuous sequence.

The evidence for glacier-lakes in the lower parts of Eskdale, Miterdale, and Wasdale is based upon overflow-channels—some marginal to the ice, others true spillways—lake deltas and beaches, laminated sand and clays, and miscellaneous fluvio-glacial and lacustrine deposits.

Lakes Miterdale and Wasdale were formed at various levels on the north side of Muncaster Fell, across which they drained southward into Lake Eskdale, which in its turn was stabilized at different levels by the marginal channels cut in the granite shelf south of Eskdale, described by the Author in 1912.

The stages in the formation of the lakes and their deposits are described, and the positions of the ice-fronts at different times are indicated. Of the deposits, special reference is drawn to beaches, especially those fringing islands, to normal lake-deltas, and to a variety referred to as 'scale-deltas'. A fine series of moraines was accumulated in water at the snout of the Wasdale Glacier.

Some notes are added upon certain deposits of different character and composition which occur in part of the area covered by Lakes Wasdale and Miterdale, and which are attributed to a later episode (the third)—that of the readvance of Scottish ice. This ice apparently invaded the low ground, and laid down near its termination a large spread of sand and gravel, hummocky in places, but usually with a flattish surface elsewhere. The Scottish ice ponded up in Lower Miterdale a low-level lake which may first have drained away into Eskdale at the north-eastern end of Muncaster Fell, but later had an exit near the sea-coast. Laminated clays mark the site of this expanse of water.

2. 'The "One-Hundred Foot" Raised Beach between Arundel and Chichester.' By the Rev. Joseph Fowler, M.A., F.G.S.

Prestwich, in 1858, assumed that the 15-foot raised beach at Brighton is represented by the 100-foot raised beach west of Arundel. Clement Reid seems to accept this identification. There is, however, no proof, either of (1) any differential movement in the general process of elevation, or (2) a fault between the two series that might account for the discrepancy in levels. Moreover, the 'rounded shingle', cited by Prestwich as connecting the two series of marine material, is almost certainly of Tertiary age.

It seems safer to assume the presence of two stages of raised beach—a '100-foot' and a '15-foot' beach. Clement Reid appears really to support this view when he is considering the different ages of the Selsey deposits. So also, more recently, does H. J. O. White. Nevertheless, it must be noticed that there is, apparently, no ascertained example of the 100-foot beach east, or of the 15-foot beach west, of the Arun Valley.

The occurrence of a patch of beach cobbles at Slindon, 130 feet O.D., may register a third 'moment'—and the point of greatest submergence—in the Pleistocene marine episode.

The discovery of unabraded St. Acheul type hand-axes in Coombe Rock, at the surface of the 100-foot raised beach, may be significant in dating these deposits; and the occurrence of the 100-foot beach material in close relationship with both the Coombe Rock deposits and Chalk dry valleys, may help to throw light on the sequences of these several Pleistocene phenomena.

Prof. J. W. GREGORY, LL.D., D.Sc., F.R.S., V.P.G.S., exhibited and commented upon a chart showing a newly-discovered 700-foot depression on the floor of the North Sea. He said that the Hydrographer to the Admiralty had drawn his attention to the discovery of a series of narrow depressions on the floor of the North Sea, of which one was 130 fathoms deep, in an area where the charts showed an undulating plain of 40–50 fathoms deep. The deepest had such a steep slope that it had occasioned loss of trawling gear, and had been given the name of the 'Devil's Hole'. This deep was one of a series trending north and south near the meridian of Greenwich, and extending from about 60 miles east of Berwick-on-Tweed to 100 miles east of Montrose. An earthquake on February 5th, 1927, affected the coast of Scotland as far south as the Tay: but the trend of the isoseists and the position of the epicentre (according to the evidence collected by Dr. G. W. Tyrrell) showed that these depressions were not due to that earthquake. They might be regarded as part of the ancient valley of the Rhine, and were in line with the northern end of the pre-glacial Rhine valley, as shown by the speaker in a map published in 1927 (*Geogr. Journ.* vol. lxx, 1927, p. 58), and a furrow in the 50-fathom line shown on the Admiralty Charts.

The deeps were doubtless pre-glacial, and were difficult to reconcile with the idea that near their position the Scandinavian glaciers met and pushed back the Scottish glaciers.

The 'Devil's Hole' appeared to be in bare rock, as the grab-sounder sunk by *H.M.S.S. Fitzroy* failed to bring up any of the sea-floor.

The details were shown on the chart exhibited and in the Admiralty 'Notice to Mariners,' No. 1720, 1930.

[The Editors do not hold themselves responsible for the views expressed by their correspondents.]

FIG. 2.

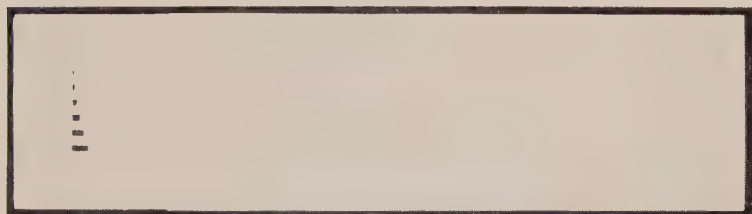


FIG. 3.

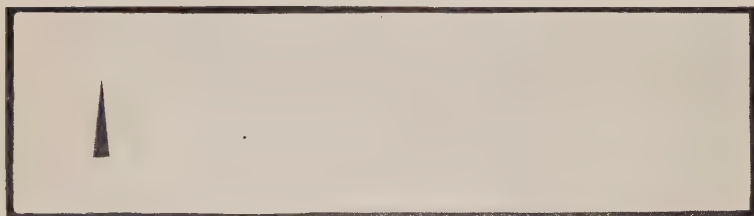


FIG. 4.

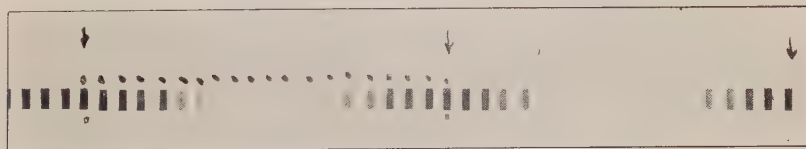
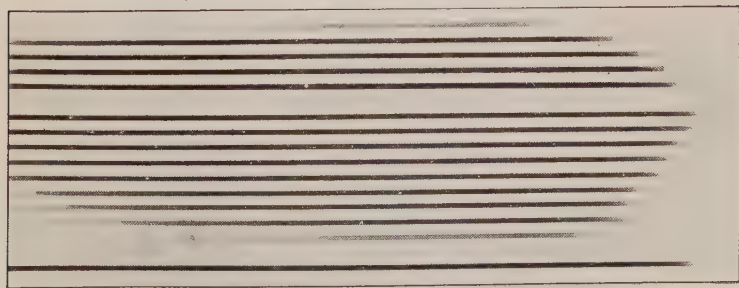


FIG. 5.





The Panelled Room.

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END OF THE ELEVENTH VOLUME.